Dynamics under location uncertainty and other energy-related stochastic subgrid schemes

V. Resseguier
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Motivations

- More rigorously identified sudgrid dynamics effects

- Quantification of modeling errors (UQ)

→ Ensemble forecasts and data assimilation

Contents

- Models under location uncertainty (LU)
- Some parameterization of the models under location uncertainty
- A new energy-budget-based stochastic scheme: WaveHyperv
- Numerical comparisons

Part I
Models under location uncertainty (LU)

LU :Adding random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved large scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \dfrac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

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$\boldsymbol{w}$

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LU :Adding random velocity

Resolved large scales

White-in-time small scales

$$\boldsymbol{v} = \boldsymbol{w} + \boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

References :

Mikulevicius & Rozovskii, 2004
Flandoli, 2011

Memin, 2014
Resseguier et al. 2017 a, b, c
Cai et al. 2017
Chapron et al. 2018
Yang & Memin 2019
Resseguier et al. 2019 a,b

Holm, 2015
Holm and Tyranowski, 2016
Arnaudon et al. 2017
Cotter and al. 2017

Crisan et al., 2017
Gay-Balmaz & Holm 2017
Cotter and al. 2018 a, b
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Large scales:

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Small scales:

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LU

Memin, 2014
Resseguier et al. 2017 a, b, c
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Chapron et al. 2018
Yang & Memin 2019
Resseguier et al. 2019 a,b

SALT

Holm, 2015
Holm and Tyranowski, 2016
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Large scales:

$\boldsymbol{w}$

Small scales:

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Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection of tracer $\boldsymbol{\Theta}$

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} \boldsymbol{d}\boldsymbol{B}\,(\boldsymbol{\sigma} \boldsymbol{d}\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma dB}\,(\boldsymbol{\sigma dB})^T\}}{dt}$$

Advection of tracer $\boldsymbol{\Theta}$

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla}\Theta \right)$$

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{B}$

Variance tensor:

$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \dfrac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Advection of tracer $\boldsymbol{\Theta}$

Advection

$$\partial_t \Theta + \boxed{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta} = \boldsymbol{\nabla} \cdot \left(\frac{1}{2} \boldsymbol{a} \boldsymbol{\nabla} \Theta \right)$$

Advection of tracer $\boldsymbol{\Theta}$

Large scales:

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Small scales:

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$$\partial_t \Theta + \underbrace{\boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}_{\text{Diffusion}}$$

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Drift correction

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B}\,(\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

Multiplicative random forcing

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla}\cdot\left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)$$

Drift correction

Large scales:

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$$\partial_t \Theta + \underbrace{\boldsymbol{w}^{\star} \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{\boldsymbol{B}} \cdot \boldsymbol{\nabla}\Theta}_{\text{Advection}} = \underbrace{\boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)}_{\text{Diffusion}}$$

Drift correction

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Advection of tracer $\boldsymbol{\Theta}$

Multiplicative random forcing

Balanced energy exchanges

Advection

Diffusion

$$\partial_t \Theta + \boldsymbol{w}^\star \cdot \boldsymbol{\nabla}\Theta + \boldsymbol{\sigma}\dot{B} \cdot \boldsymbol{\nabla}\Theta = \boldsymbol{\nabla} \cdot \left(\frac{1}{2}\boldsymbol{a}\boldsymbol{\nabla}\Theta\right)$$

Drift correction

Part II
Some parameterization of LU models

Part II

Some parameterization of LU models

$$\boldsymbol{\sigma} = ?$$

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Small scales:

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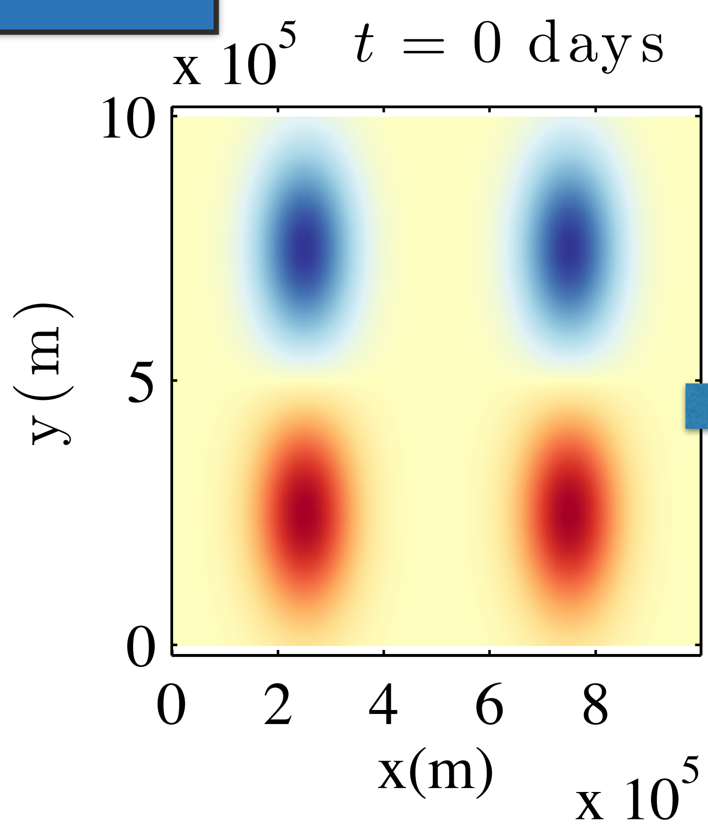
Variance tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} dB\, (\boldsymbol{\sigma} dB)^T\}}{dt}$$

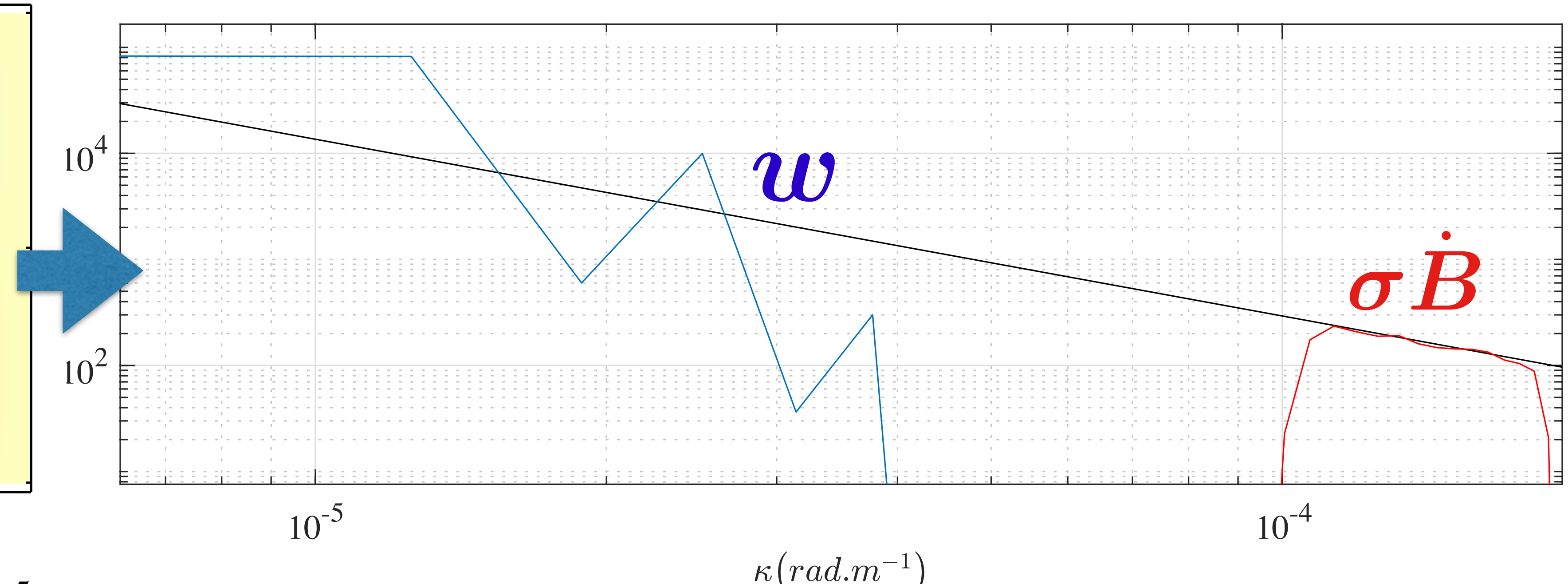
Spectral model

(homogeneous and stationary $\boldsymbol{\sigma}\dot{B}$)

MU Spec



KE Spectrum



Code online

Reference:
Resseguier, Memin & Chapron 2017b

Large scales:

$\boldsymbol{w}$

Small scales:

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Variance tensor:

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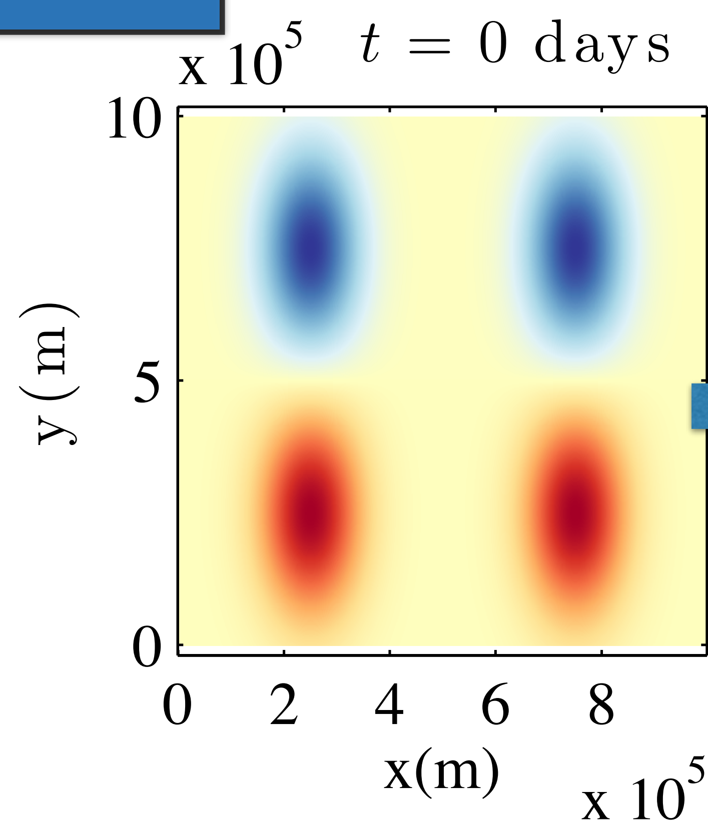
MU Spec

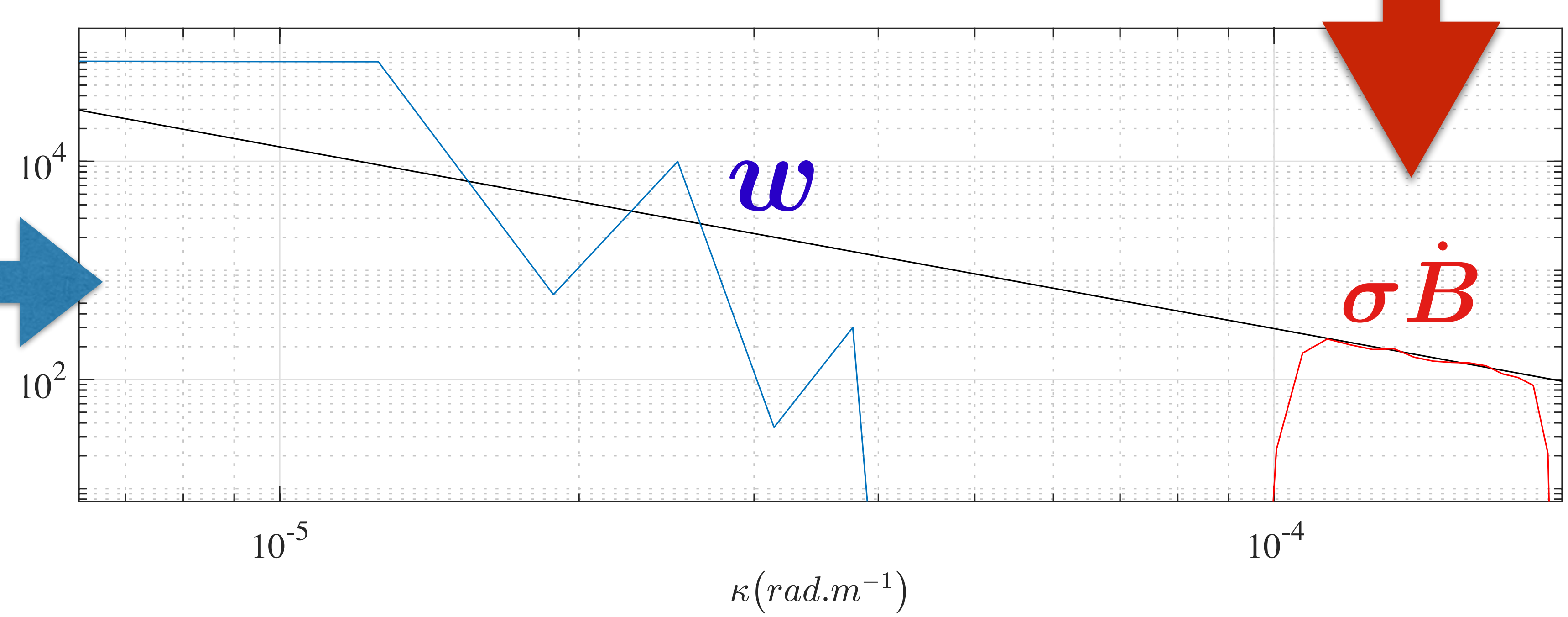
Spectral model

(homogeneous and stationary $\boldsymbol{\sigma}\dot{B}$)

Fixed spectrum at small scales



KE Spectrum



Code online

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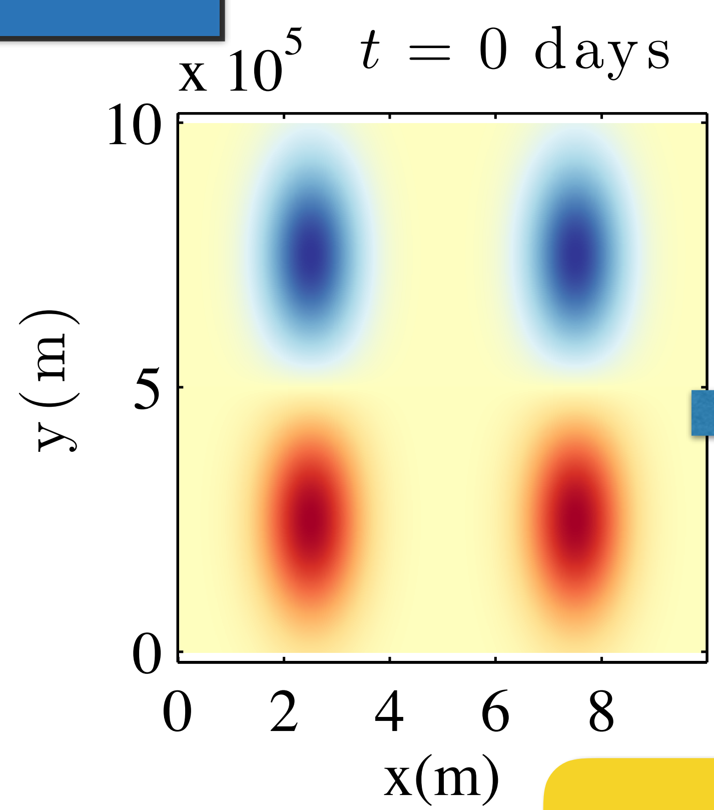
Spectral model

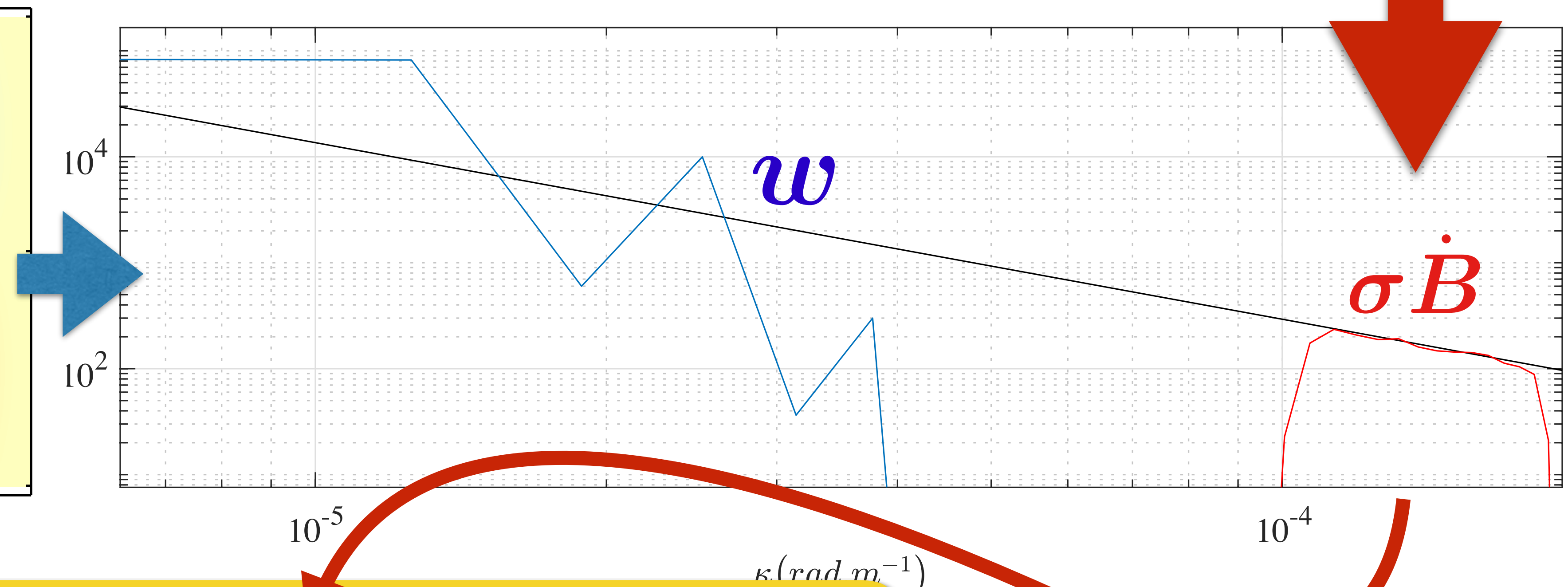
(homogeneous and stationary $\boldsymbol{\sigma}\dot{B}$)

MU Spec

Fixed spectrum at small scales



KE Spectrum



$\boldsymbol{\sigma}\dot{B} =$ (filter) $*$ (white noise)

Code online

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Large scales:

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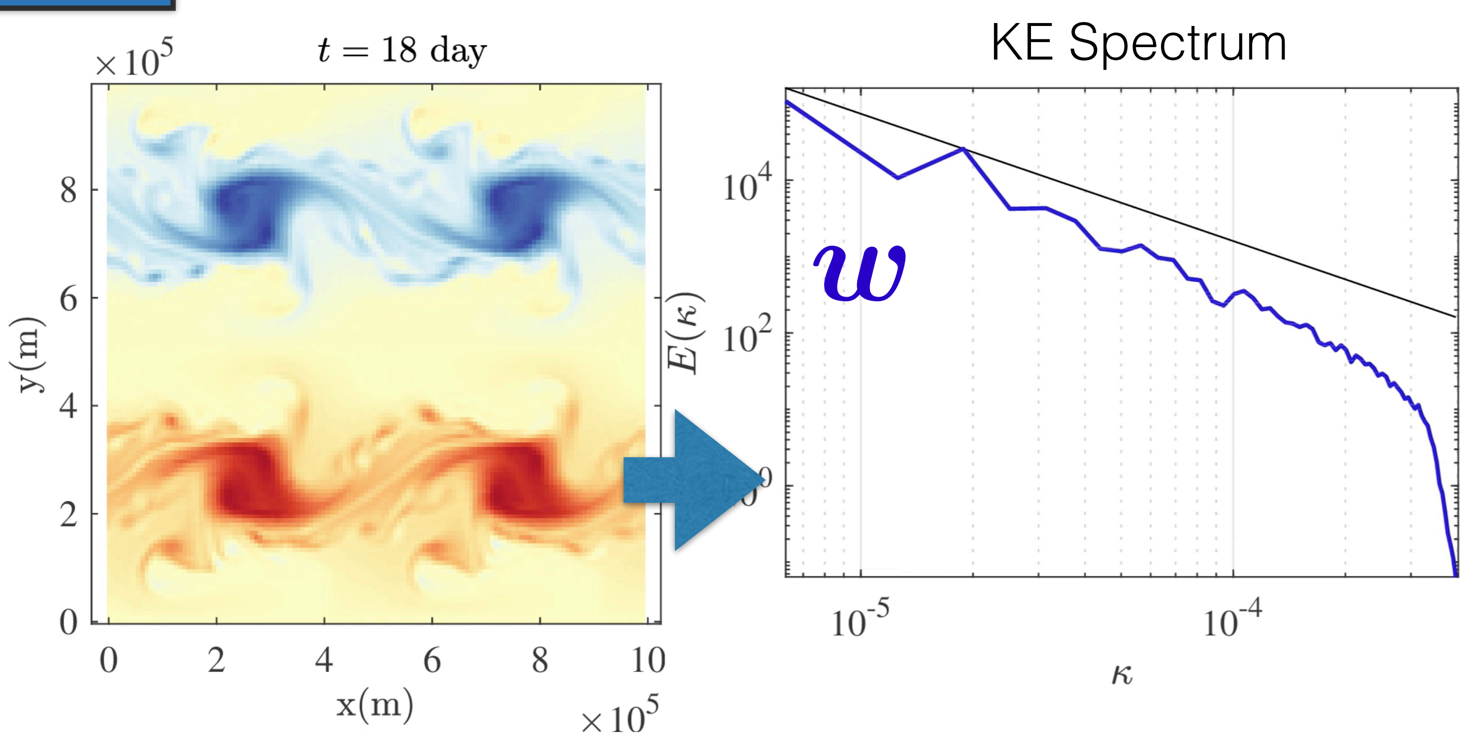
Absolute Diffusivity Spectral Density

MU ADSD

(homogeneous but non-stationary and tuning-free $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$)



Reference:
Resseguier, Pan & Fox-Kemper 2019a

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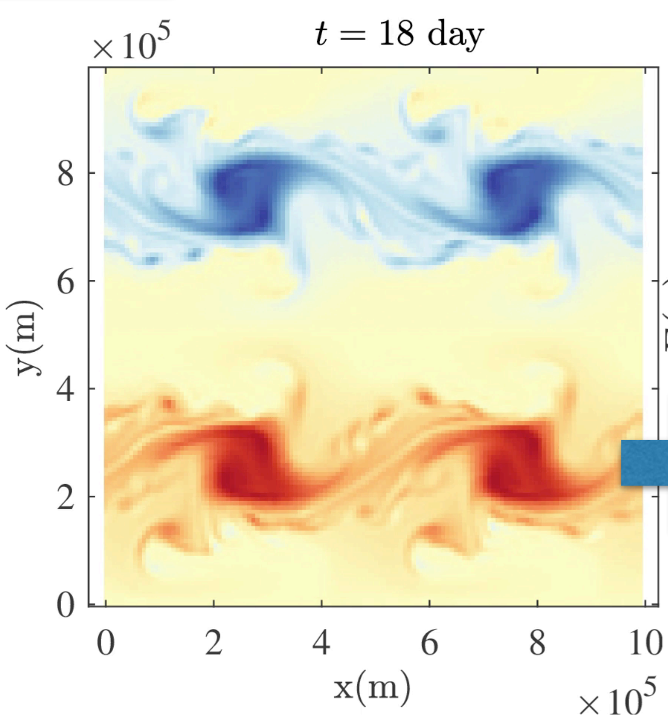
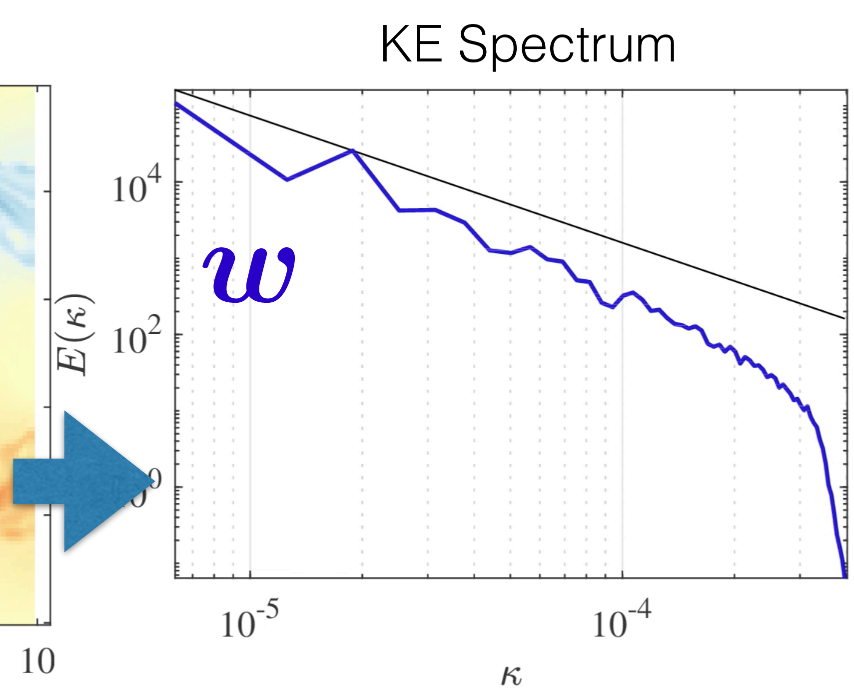
Absolute Diffusivity Spectral Density

$$A(\kappa) = E(\kappa)\tau(\kappa)$$





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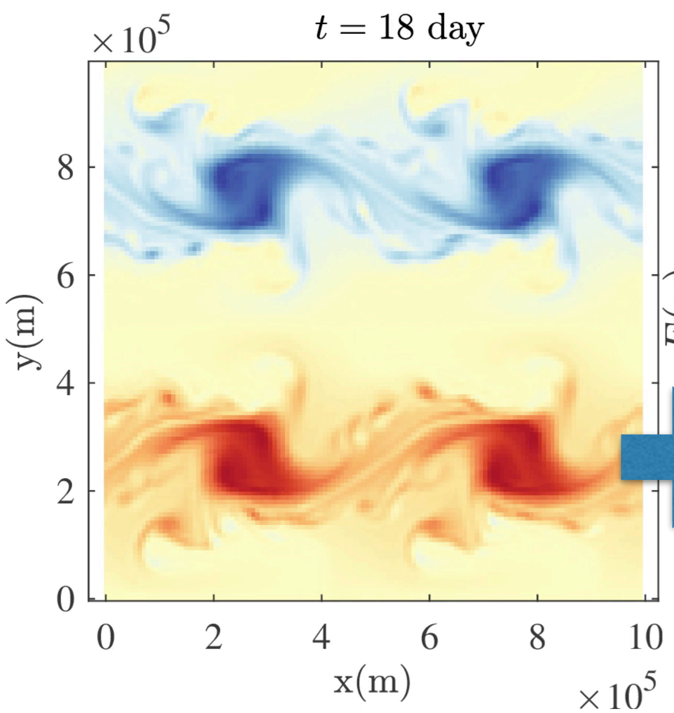

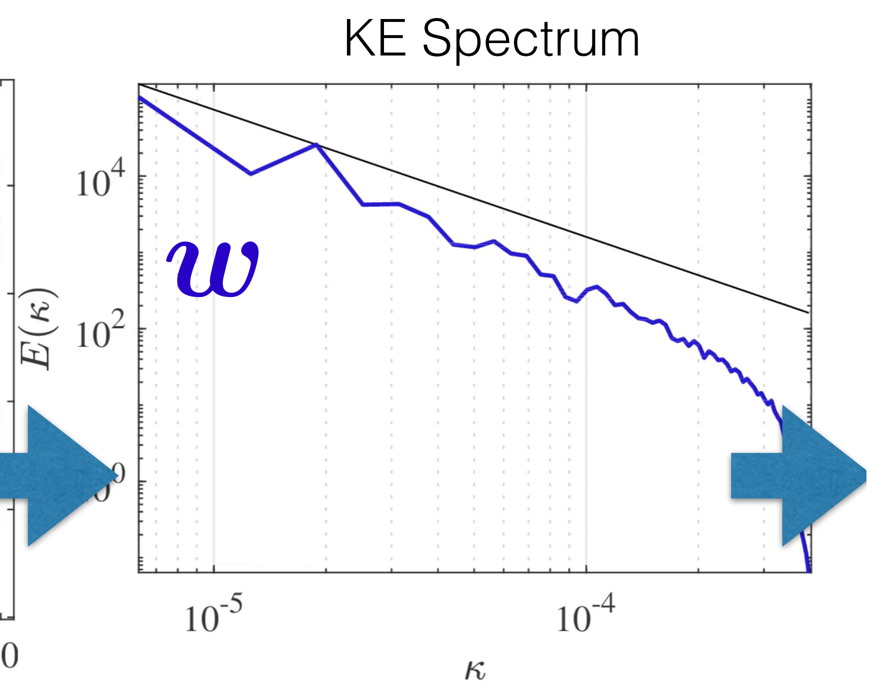

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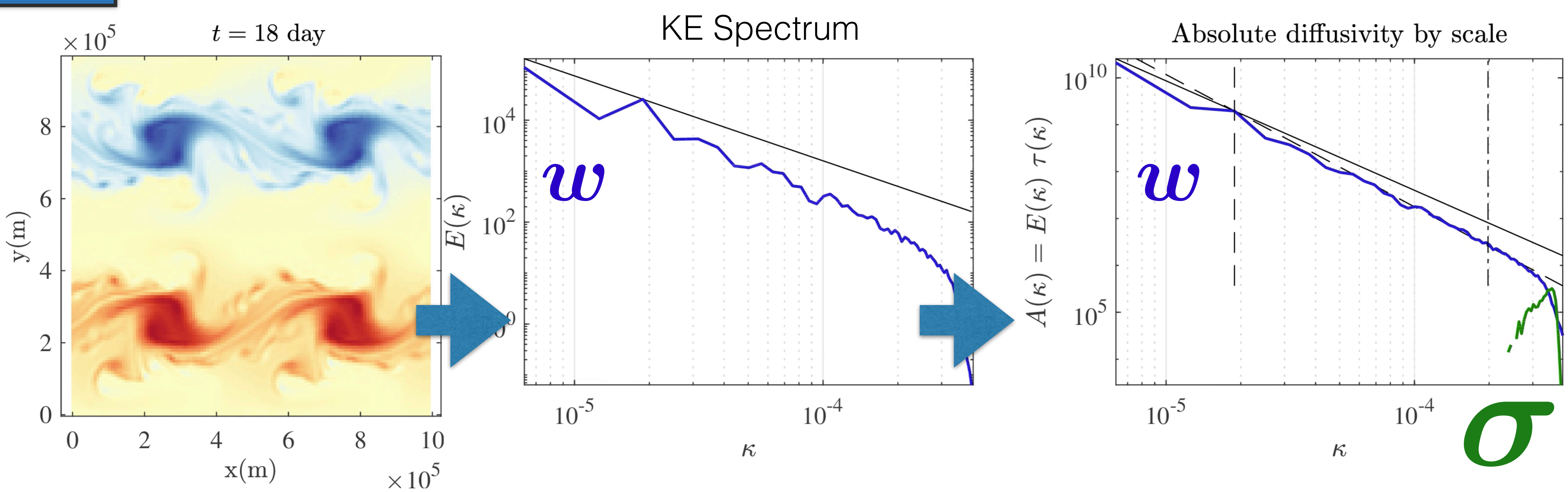

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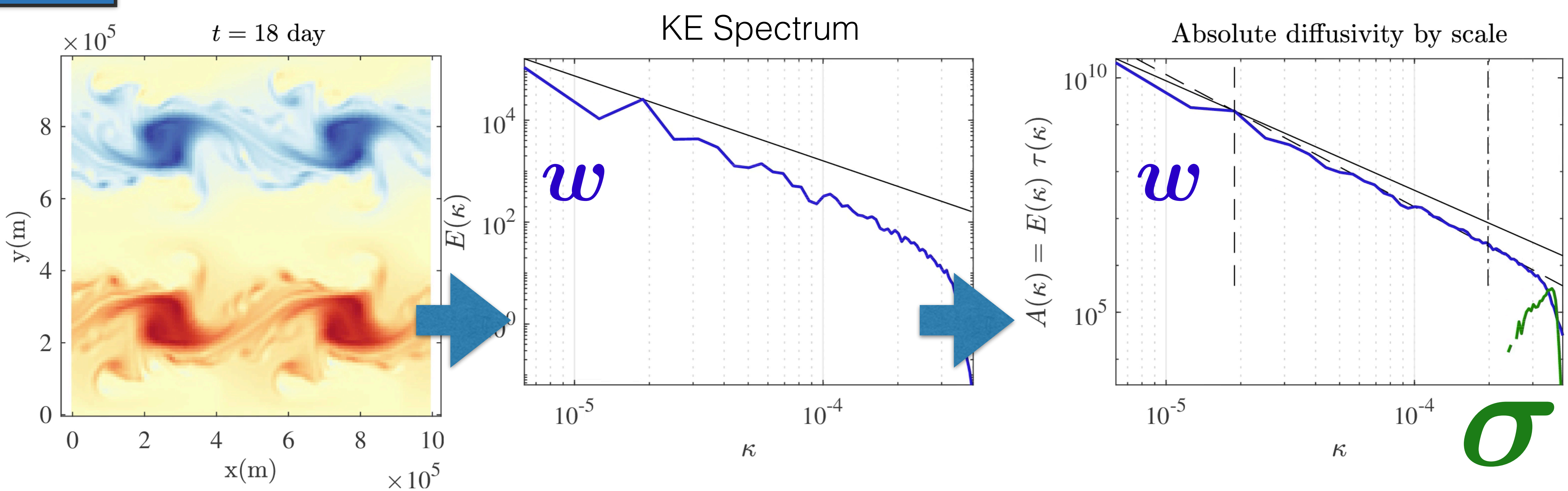

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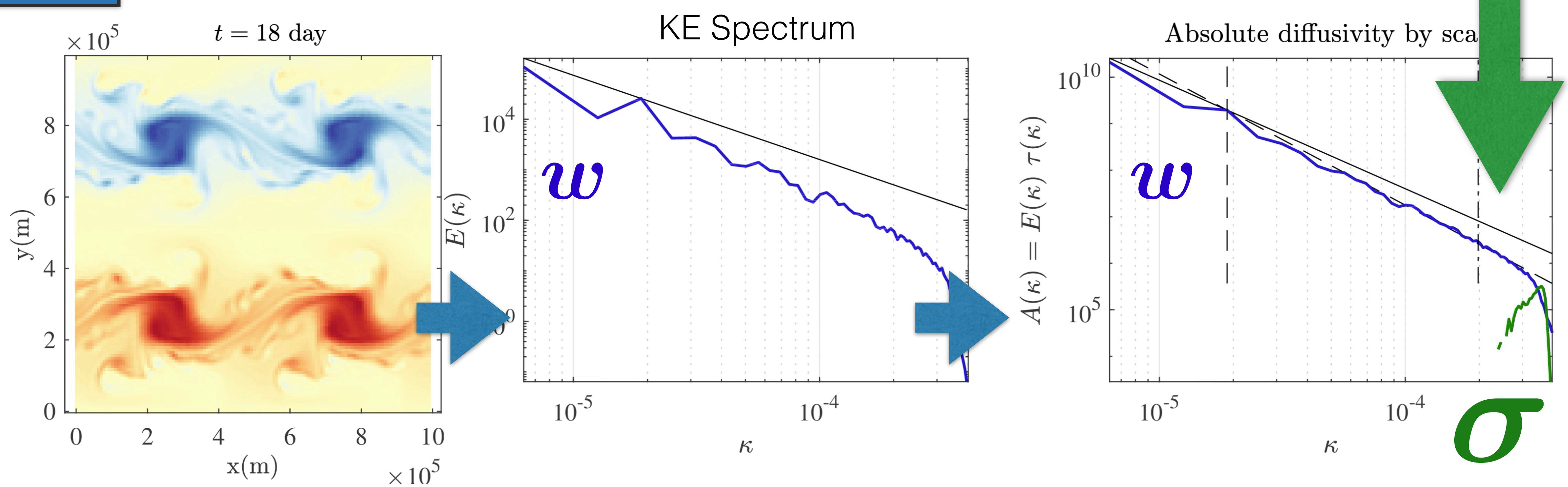
Absolute Diffusivity Spectral Density $A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



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Absolute Diffusivity Spectral Density

MU ADSD

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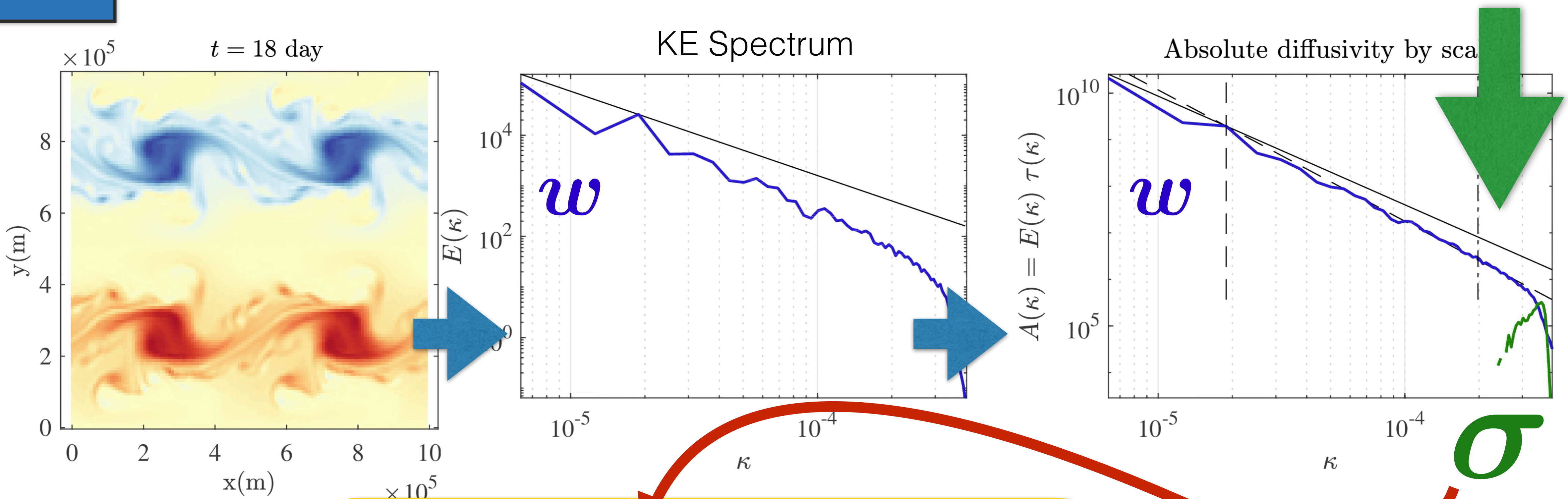
Absolute Diffusivity Spectral Density $\quad A(\kappa) = E(\kappa)\tau(\kappa)$

Residual non-stationary ADSD



$\boldsymbol{\sigma}\dot{\boldsymbol{B}} =$ (filter) $*$ (white noise)

Reference:
Resseguier, Pan & Fox-Kemper 2019a

Random switching of points

(heterogeneous and non-stationary $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$)

MU SVD

Large scales:

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Small scales:

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Variance tensor:

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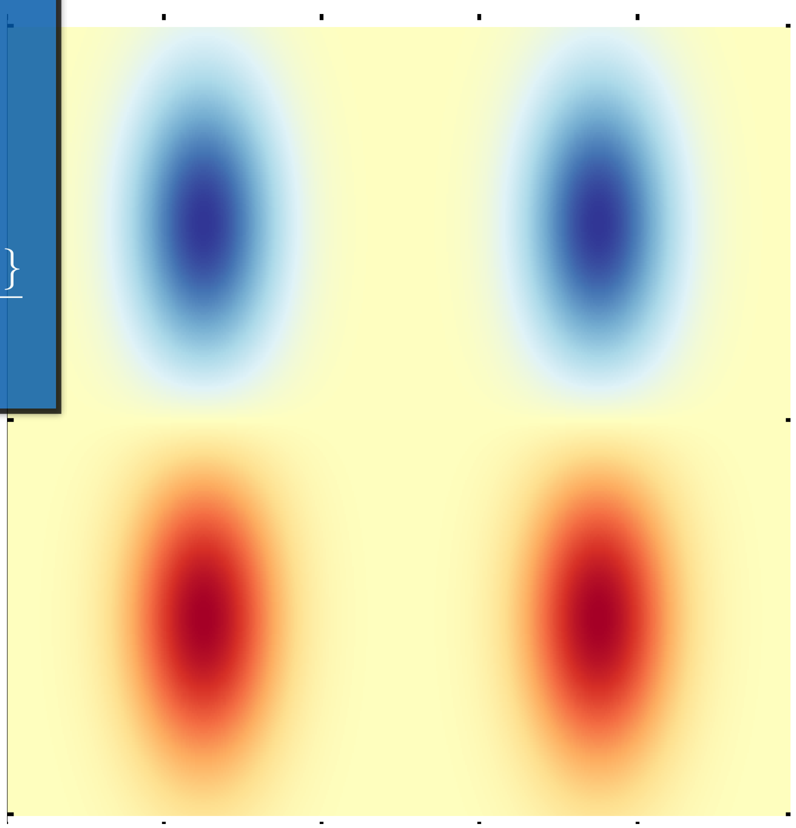

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

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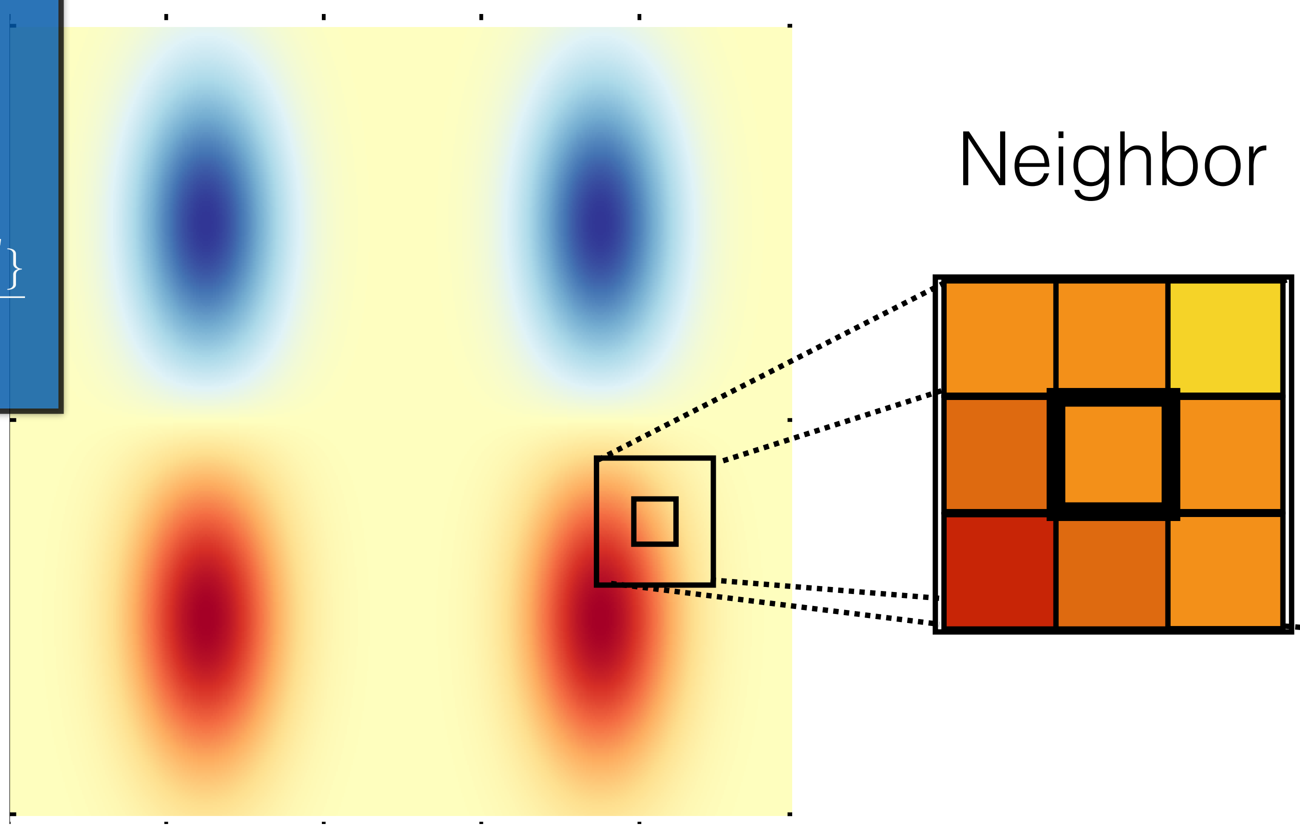

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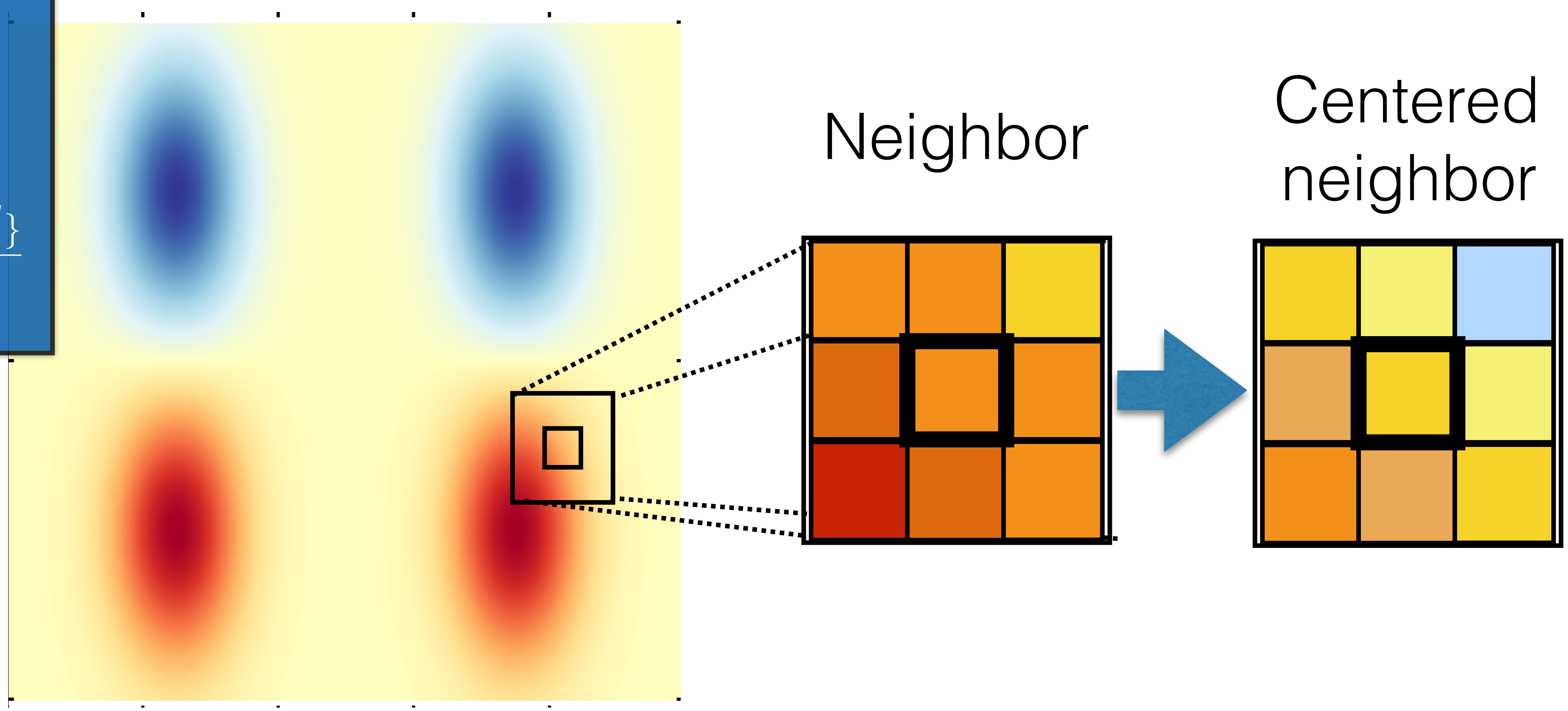

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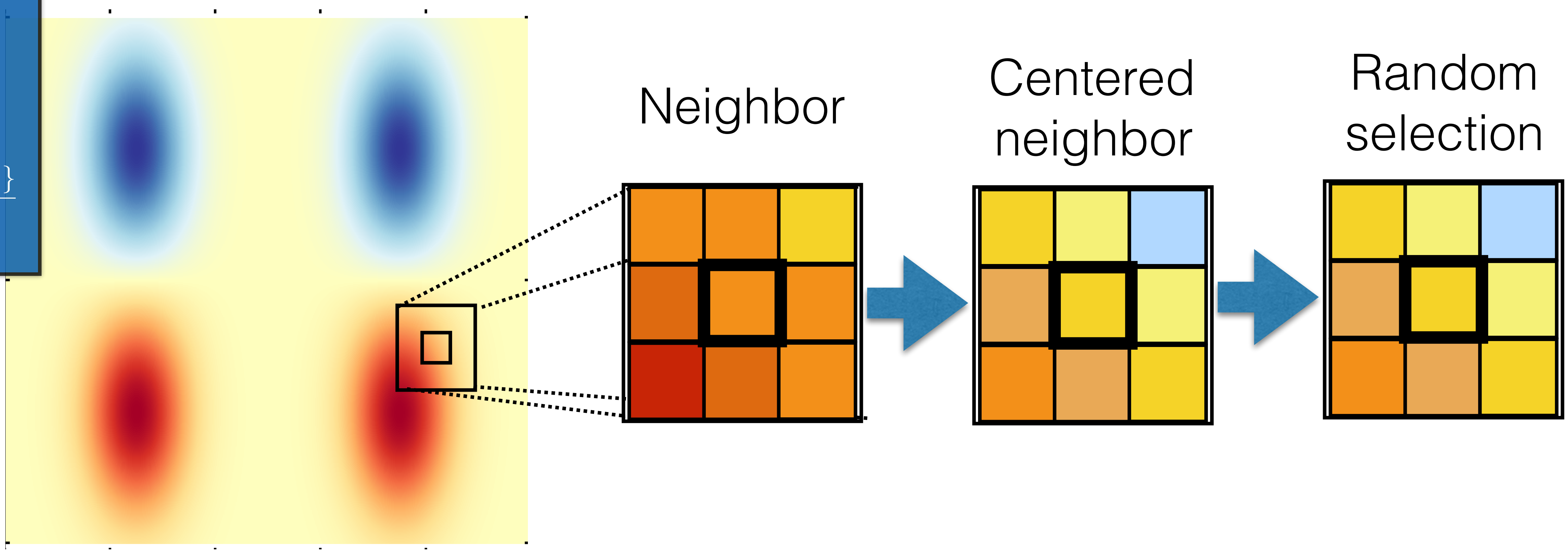

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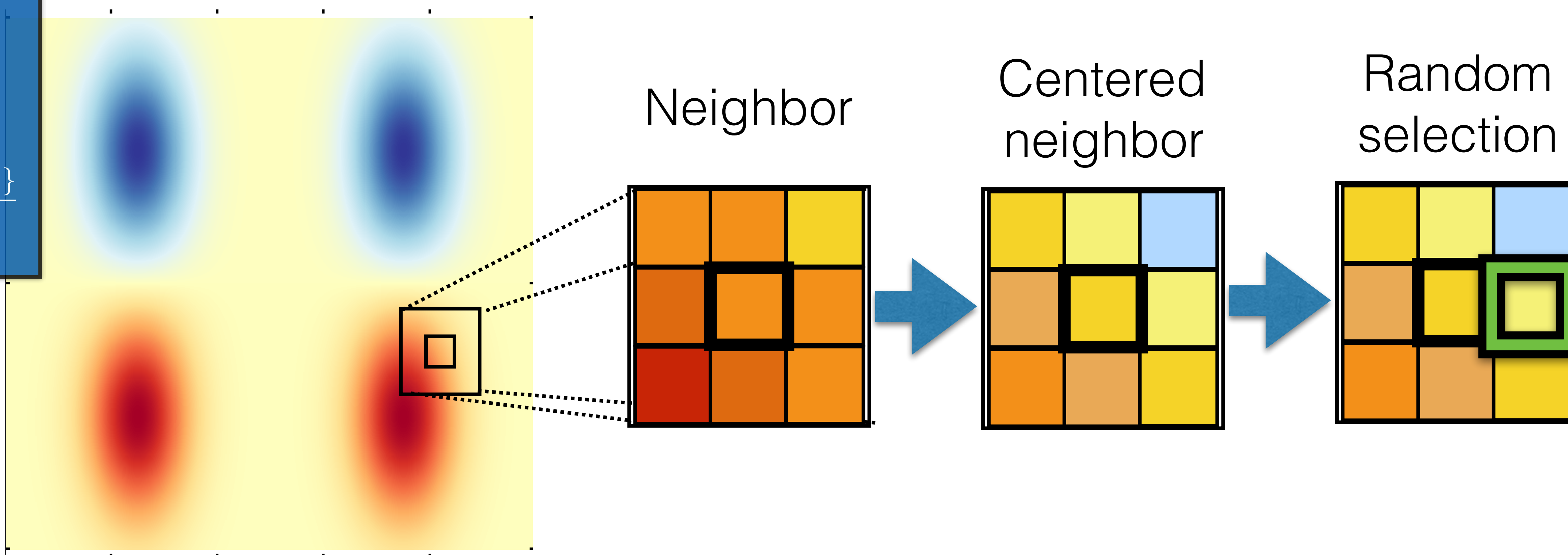

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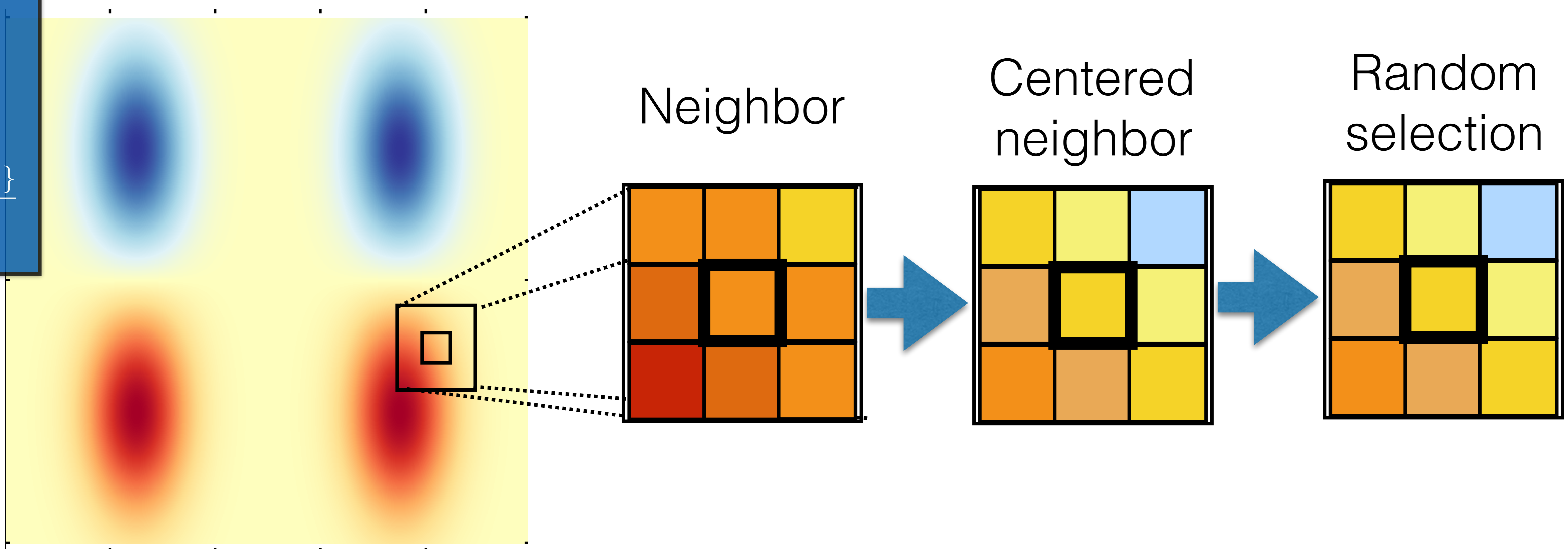

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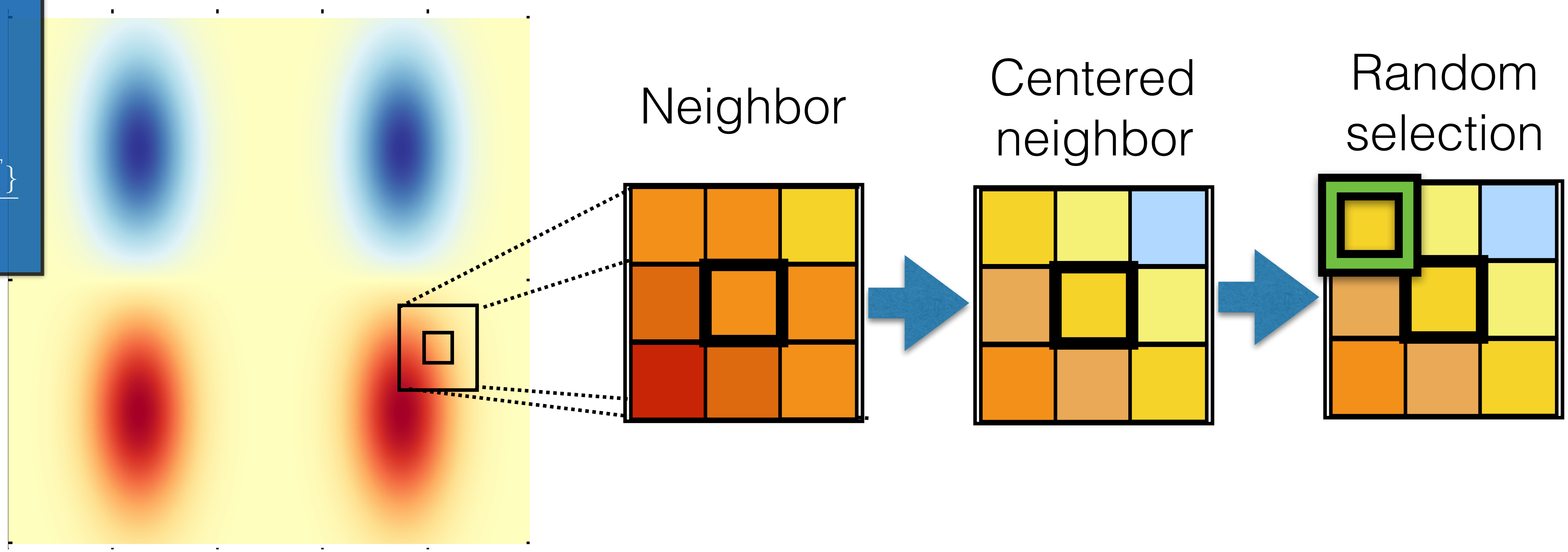

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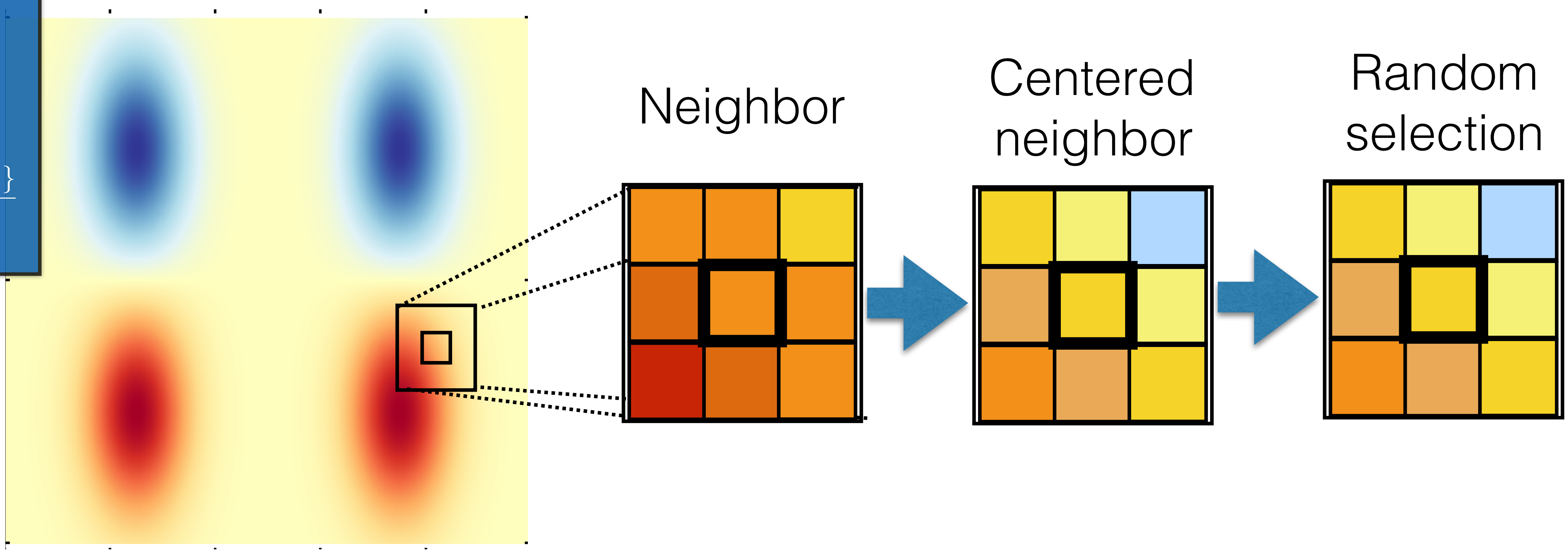

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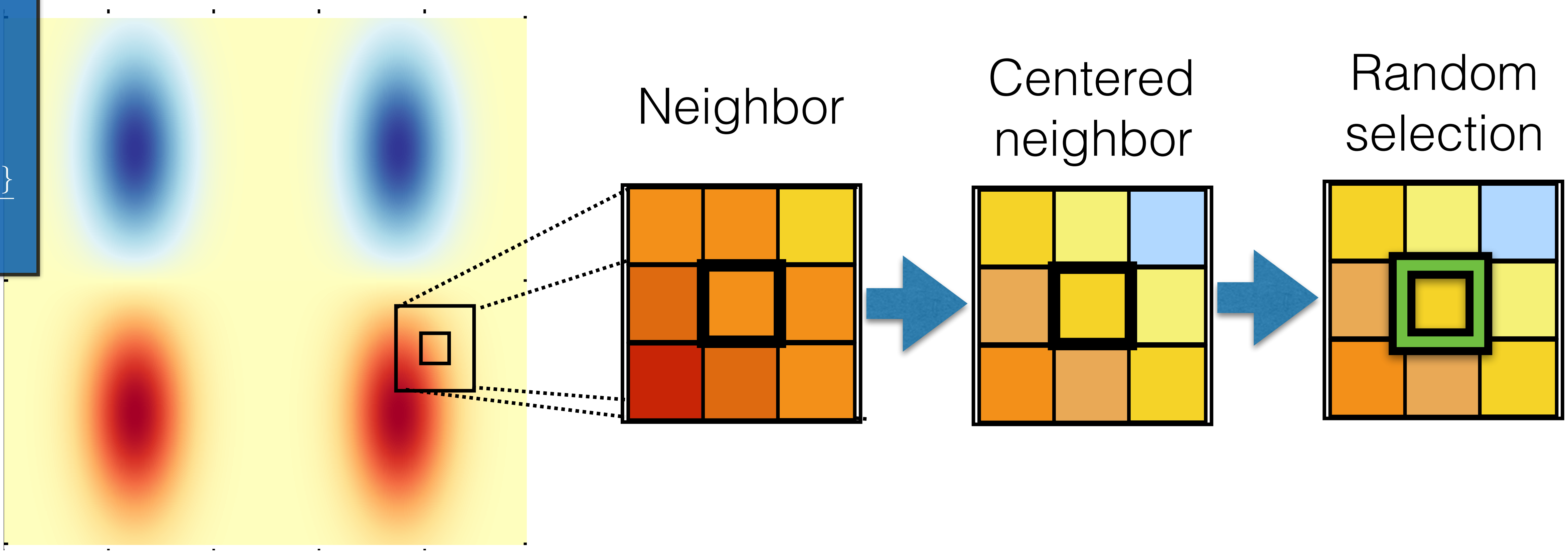
$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x},\boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} \, (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$



Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

Random switching of points

(heterogeneous and non-stationary $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$)

MU SVD

Large scales:

$\boldsymbol{w}$

Small scales:

$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$

Variance tensor:

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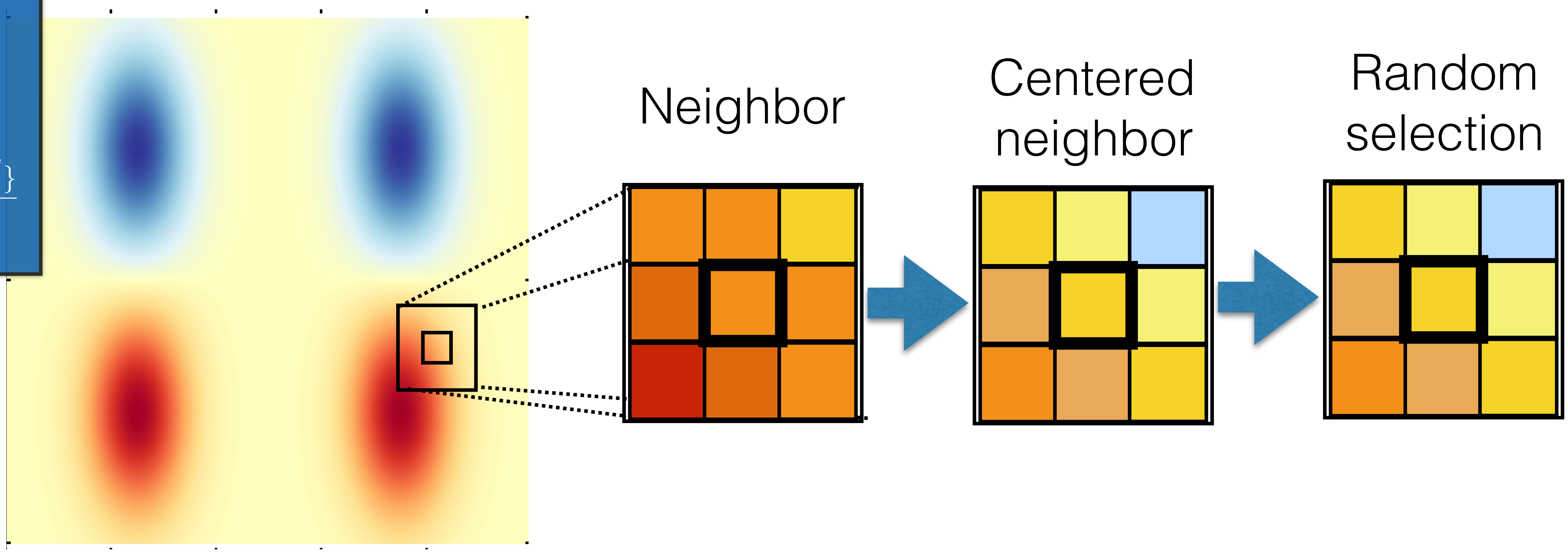

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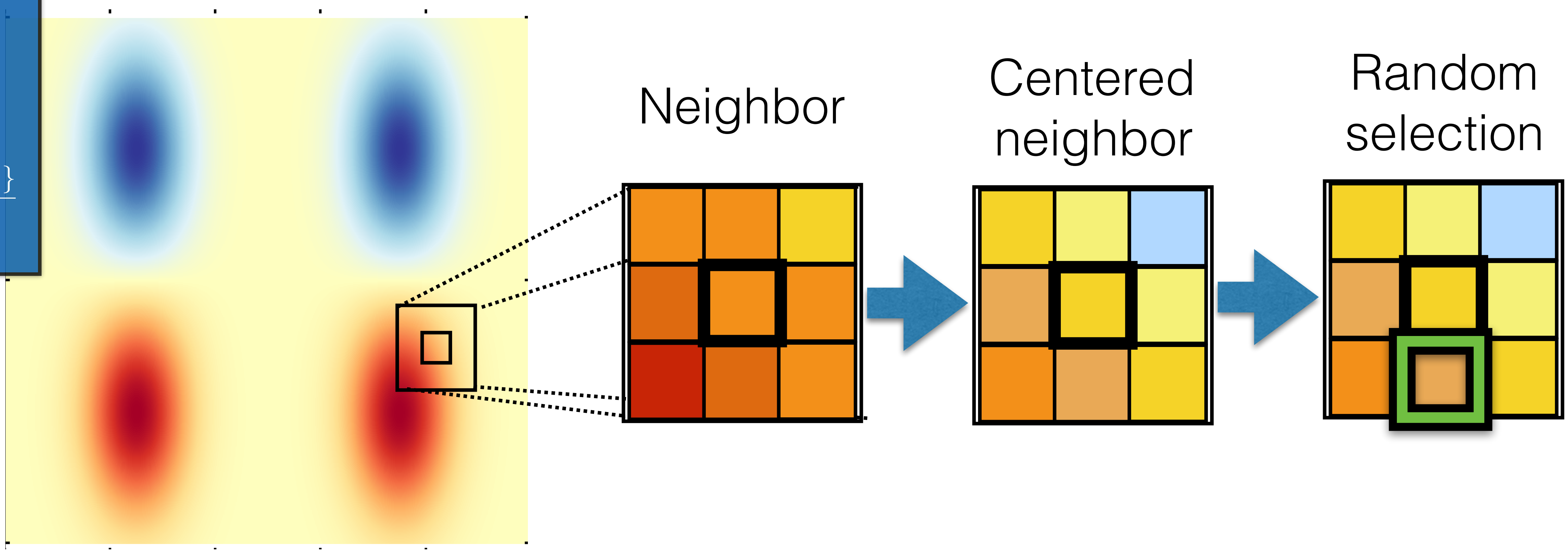

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MU SVD

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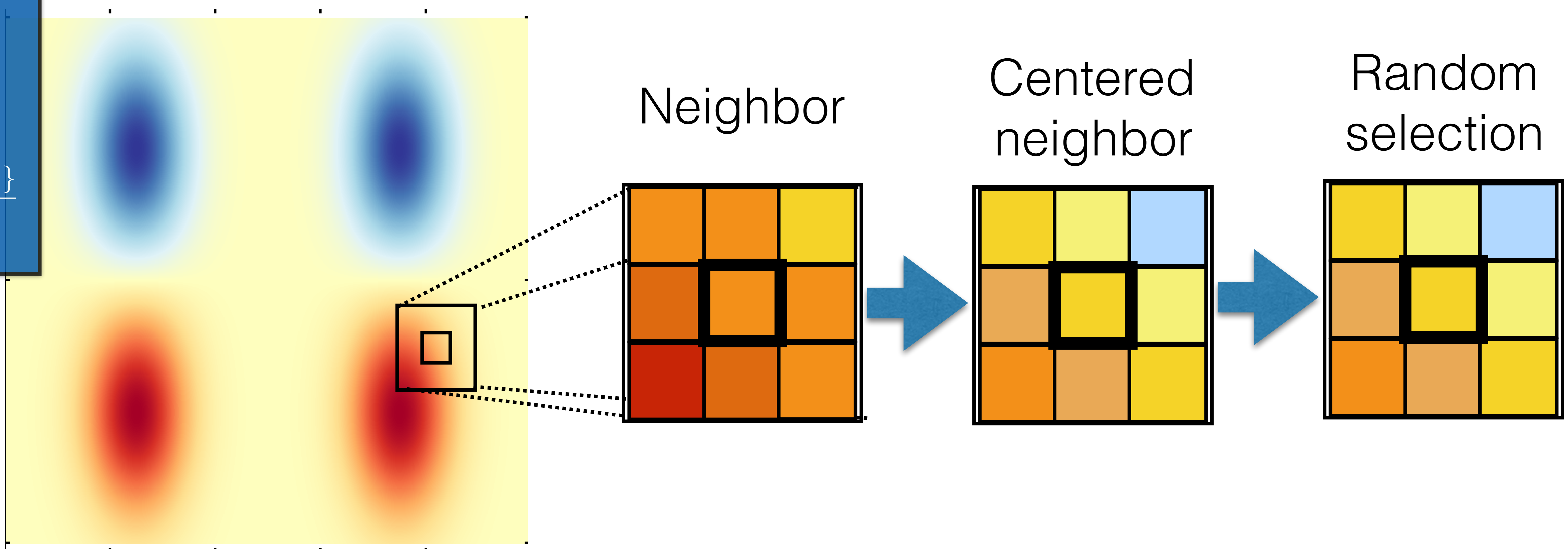

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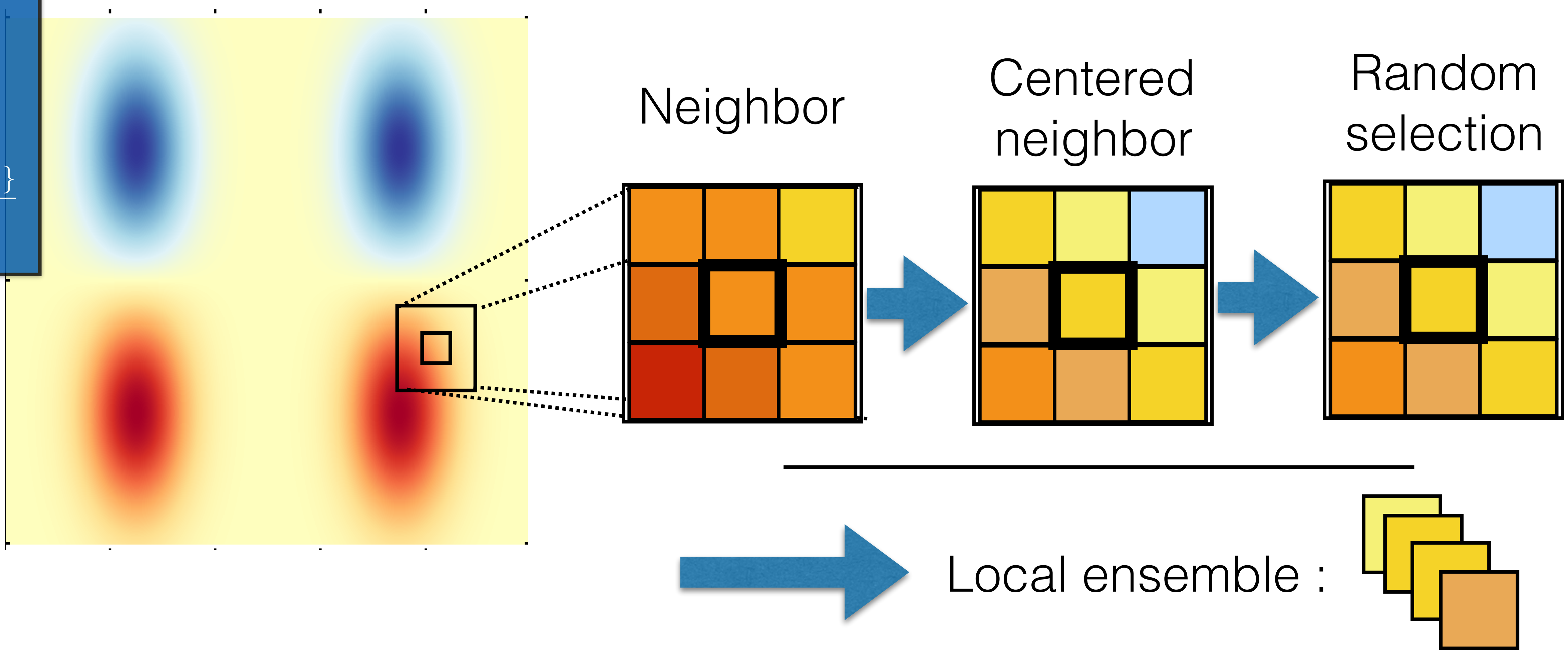

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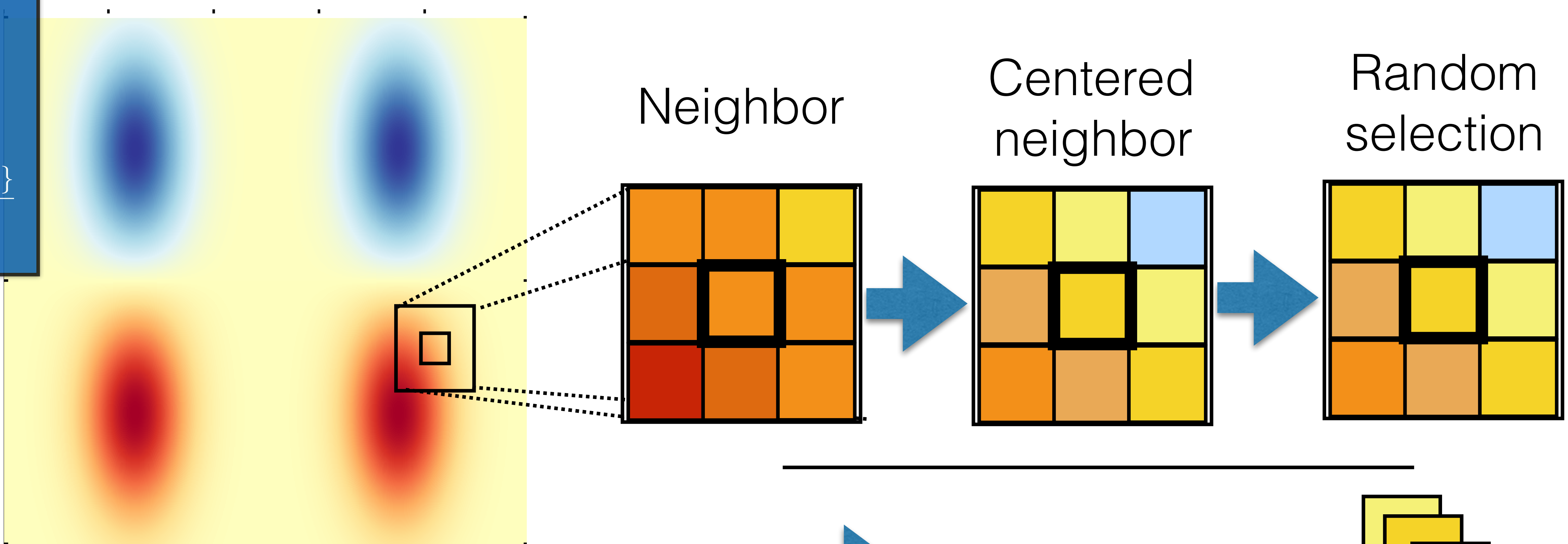
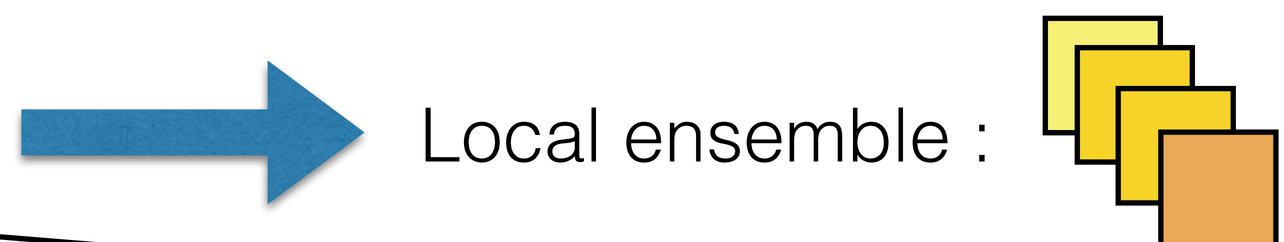
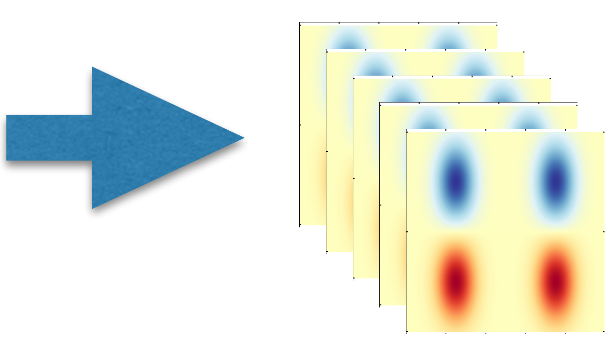




Global ensemble



Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

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(heterogeneous and non-stationary $\boldsymbol{\sigma}\dot{\boldsymbol{B}}$)

MU SVD

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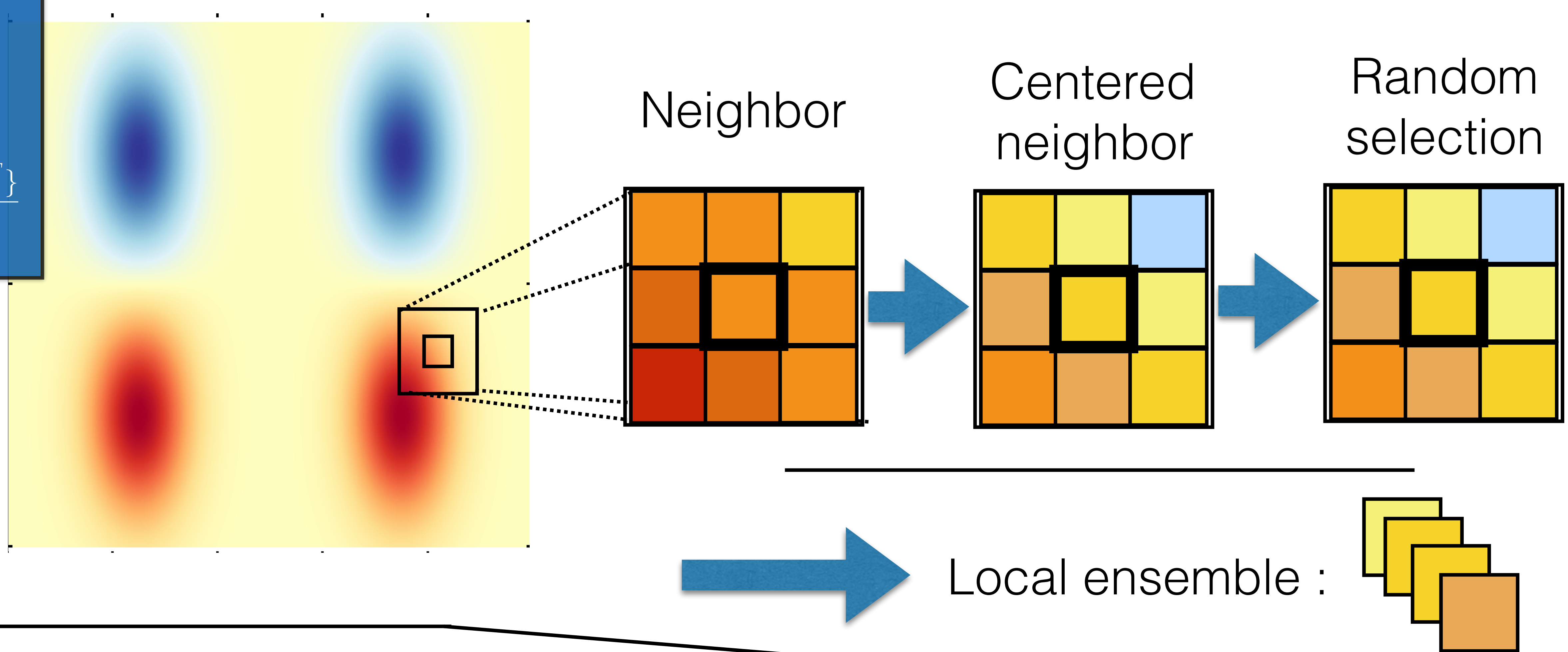
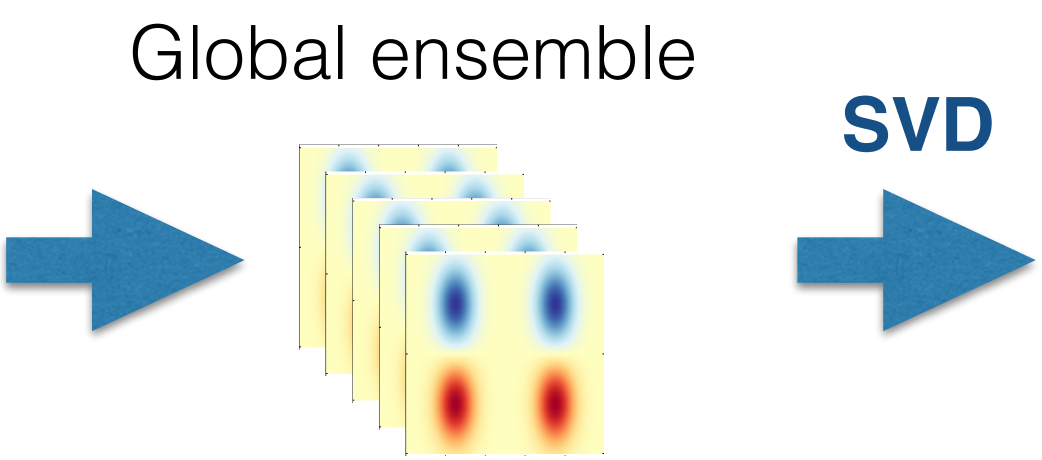




$$\boldsymbol{\sigma}\dot{\boldsymbol{B}}$$

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

Part III

A new energy-budget-based stochastic scheme:

WaveHyperv

WaveHyperv

Transport equation $$\frac{Dq}{Dt} = \mathcal{L}[q] + \eta$$

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

WaveHyperv

Transport equation $$\frac{Dq}{Dt} = \boxed{\mathcal{L}[q]} + \eta$$

Usual deterministic subgrid tensor

e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

Reference:
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

WaveHyperv

Transport equation $\frac{Dq}{Dt} = \boxed{\mathcal{L}[q]} + \boxed{\eta}$

Usual deterministic subgrid tensor

e.g. Hyper-viscosity

$$\mathcal{L}[q] = -\nu \Delta^4 q$$

Random forcing

built to meet the energy budget:

(Random energy intake) = ζ x Dissipation

<u>Reference:</u>
Resseguier, Li, Jouan, Derian, Memin & Chapron 2019b

Summary of UQ methods

Name	Method
MU Spec	LU with homogeneous and stationary small-scale velocity
MU ADSD	LU with homogeneous, non-stationary and tuning-free small-scale velocity
MU SVD	LU with inhomogeneous and non-stationary small-scale velocity
WaveHyperv	Energy-budget-based stochastic scheme
PIC Spec	Perturbed initial conditions with homogeneous noise
PIC SVD	Perturbed initial conditions with inhomogeneous noise

Part IV

Numerical comparisons

Test case 1:

$t = 17$ days

SQG

$$\frac{Db}{Dt} = \boxed{-\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}}$$

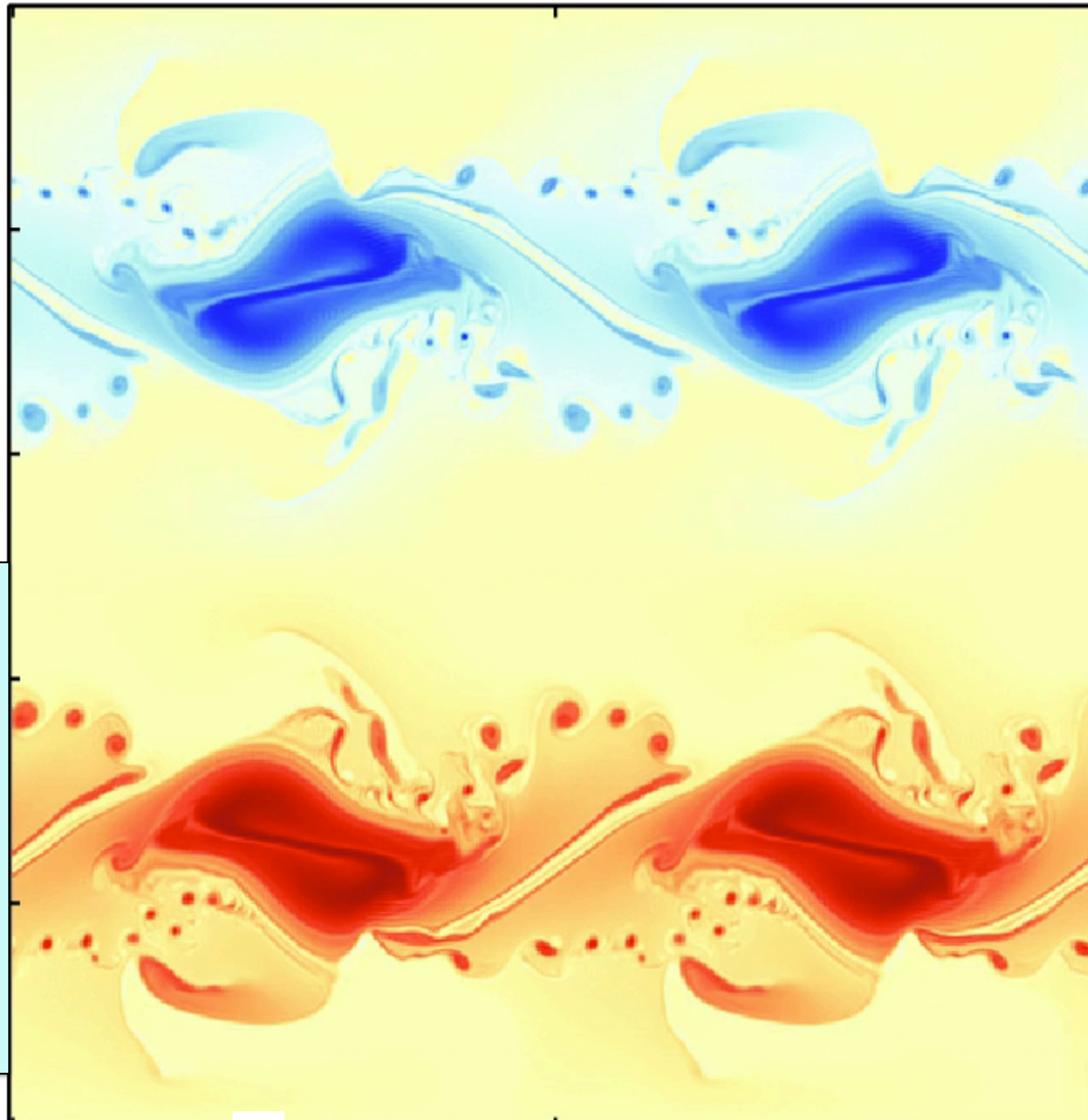
$$\boldsymbol{u} = \left(\text{cst.} \boldsymbol{\nabla}^{\perp} \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

512 x 512



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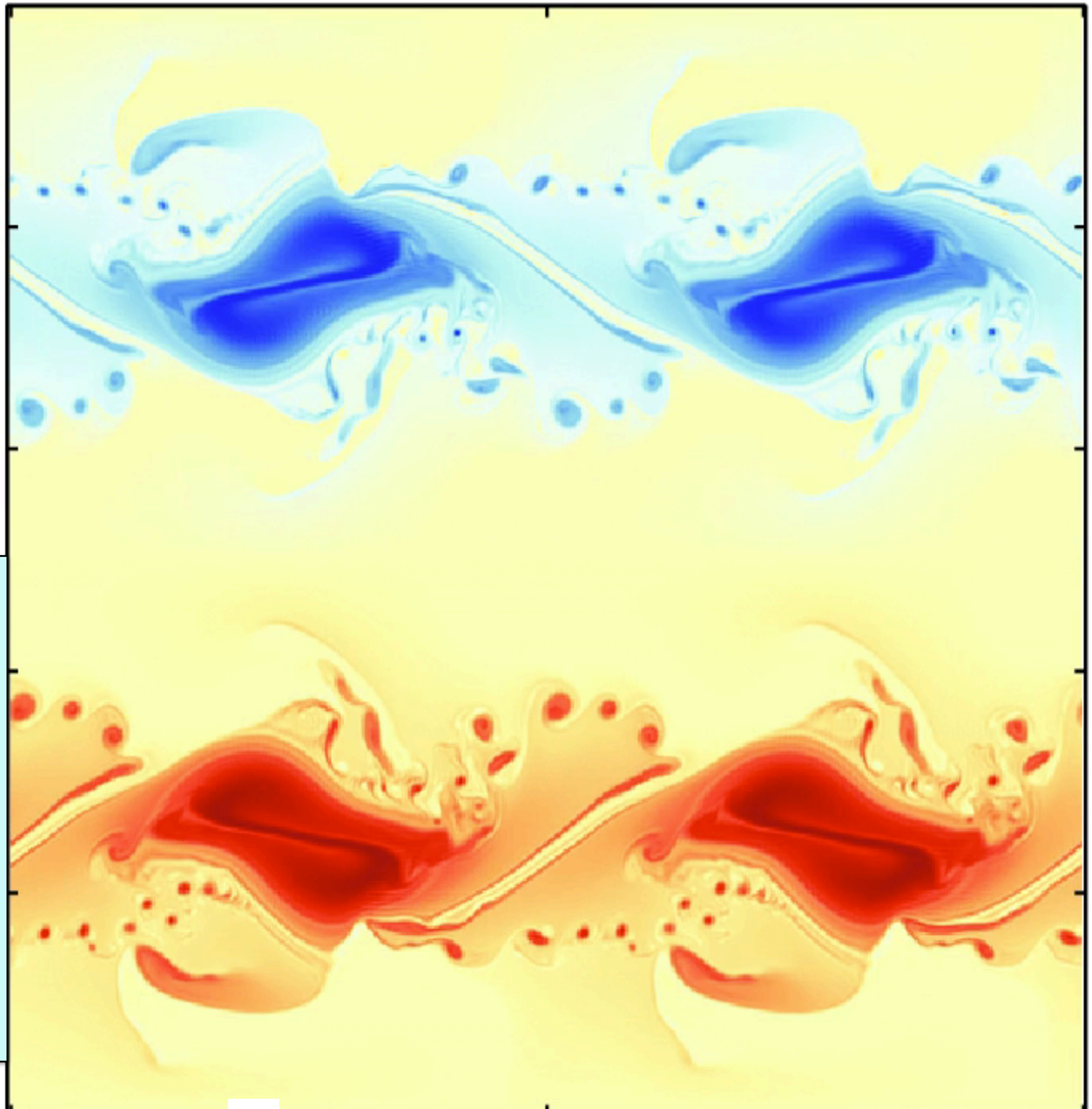


Test case 2:

SQG

$$\frac{Db}{Dt} = \boxed{-\alpha_{HV}\Delta^4 b \quad \text{Hyper-viscosity}}$$

$$\boldsymbol{u} = \left(\text{cst}.\boldsymbol{\nabla}^{\perp}\Delta^{-\frac{1}{2}}\right) b$$

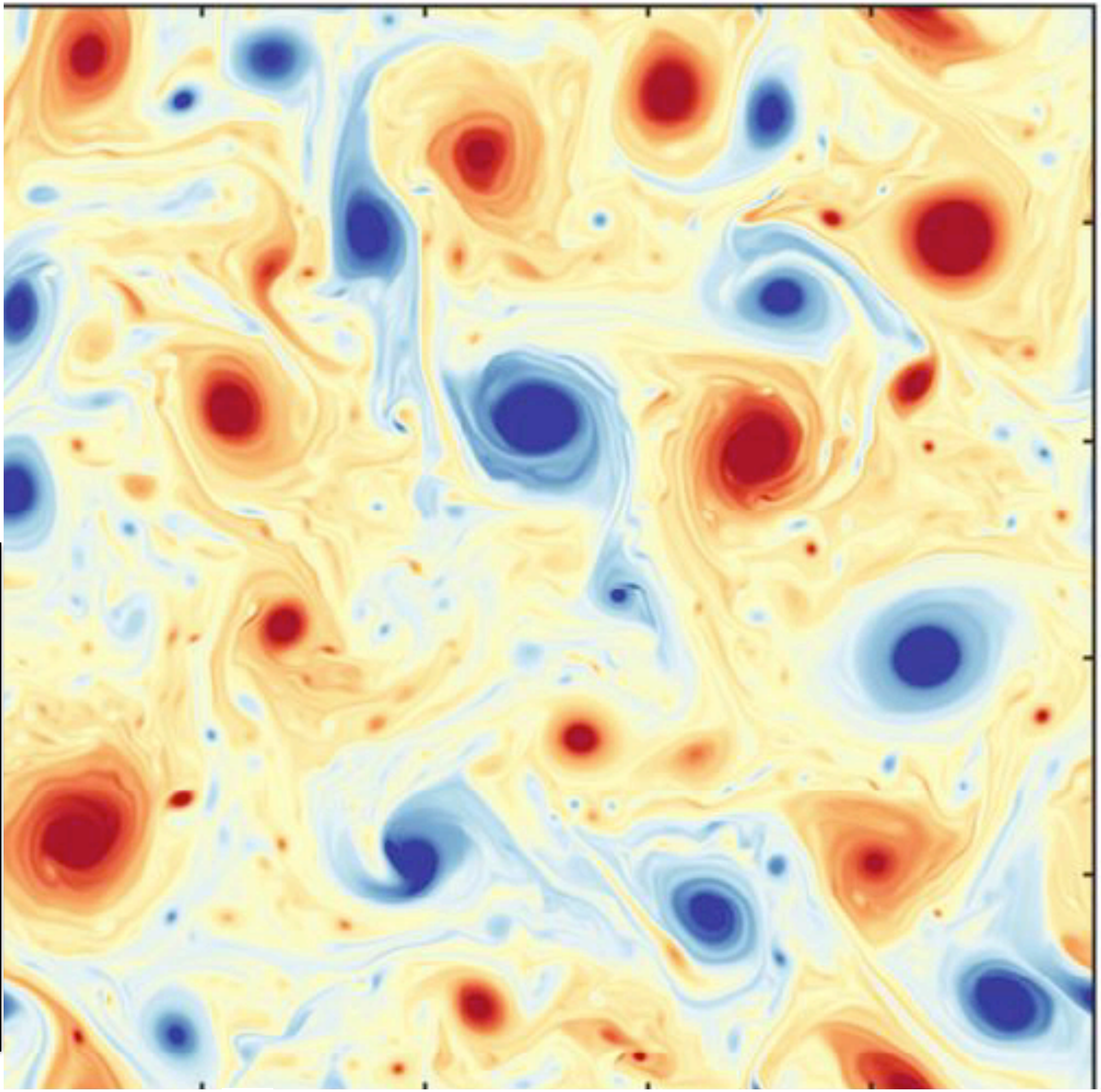
Reference flow:

deterministic

SQG

512 x 512

$t = 10$ days



Test case 2:

SQG

$$\frac{Db}{Dt} = \boxed{-\alpha_{HV}\Delta^4 b \quad \text{Hyper-viscosity}}$$

$$\boldsymbol{u} = \left(\text{cst}.\boldsymbol{\nabla}^{\perp}\Delta^{-\frac{1}{2}}\right) b$$

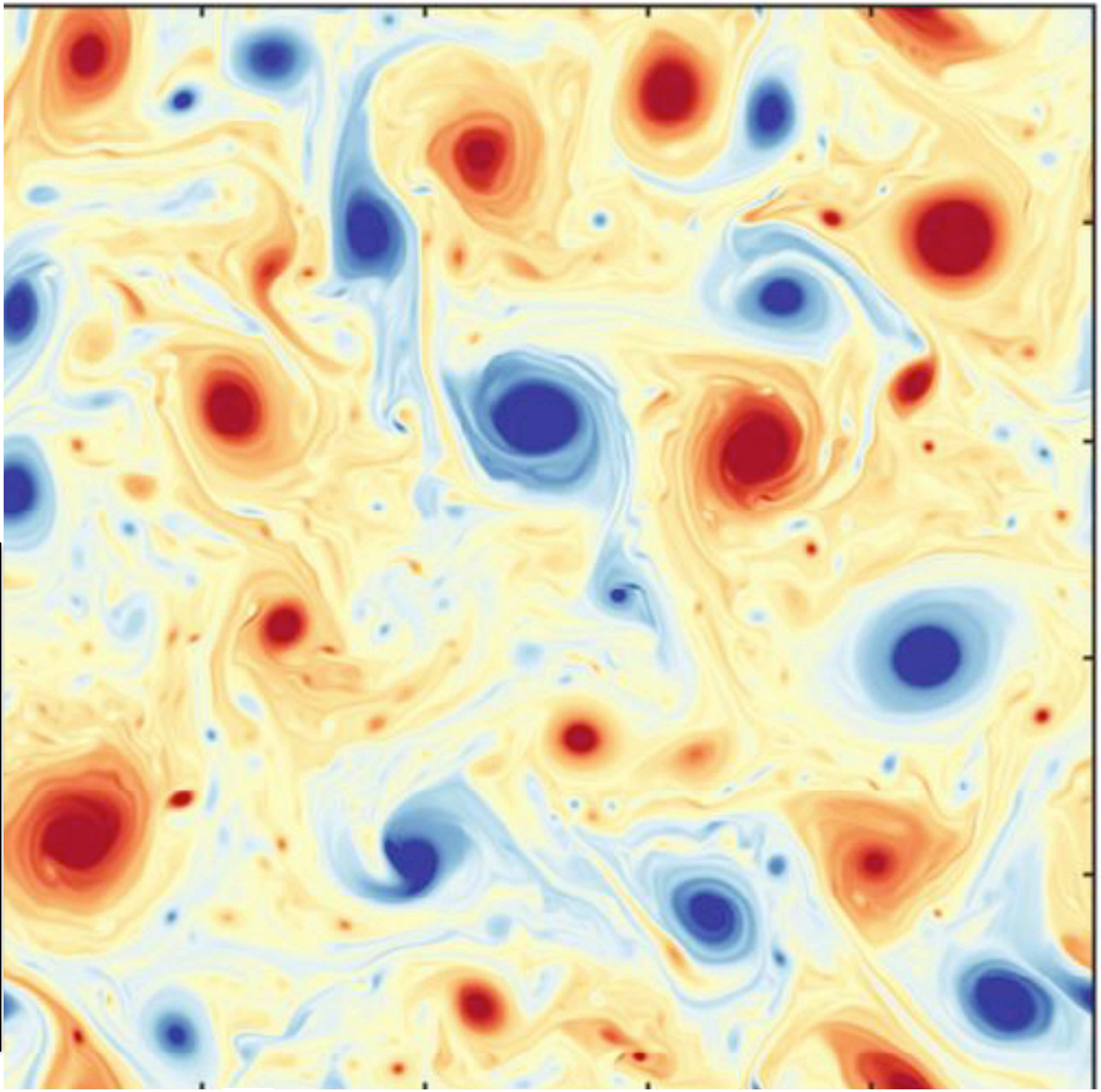
Reference flow:

deterministic

SQG

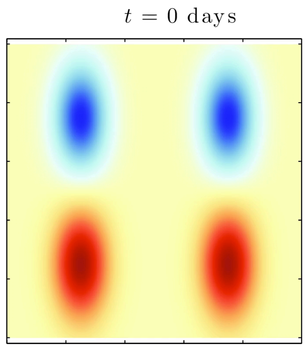
512 x 512

$t = 10$ days

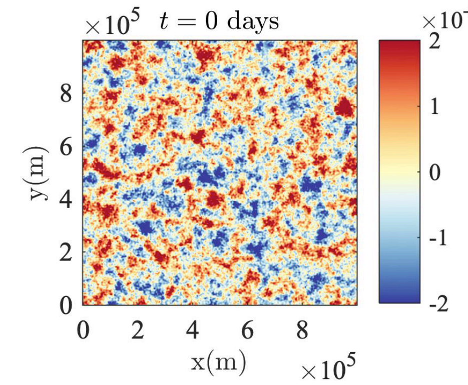


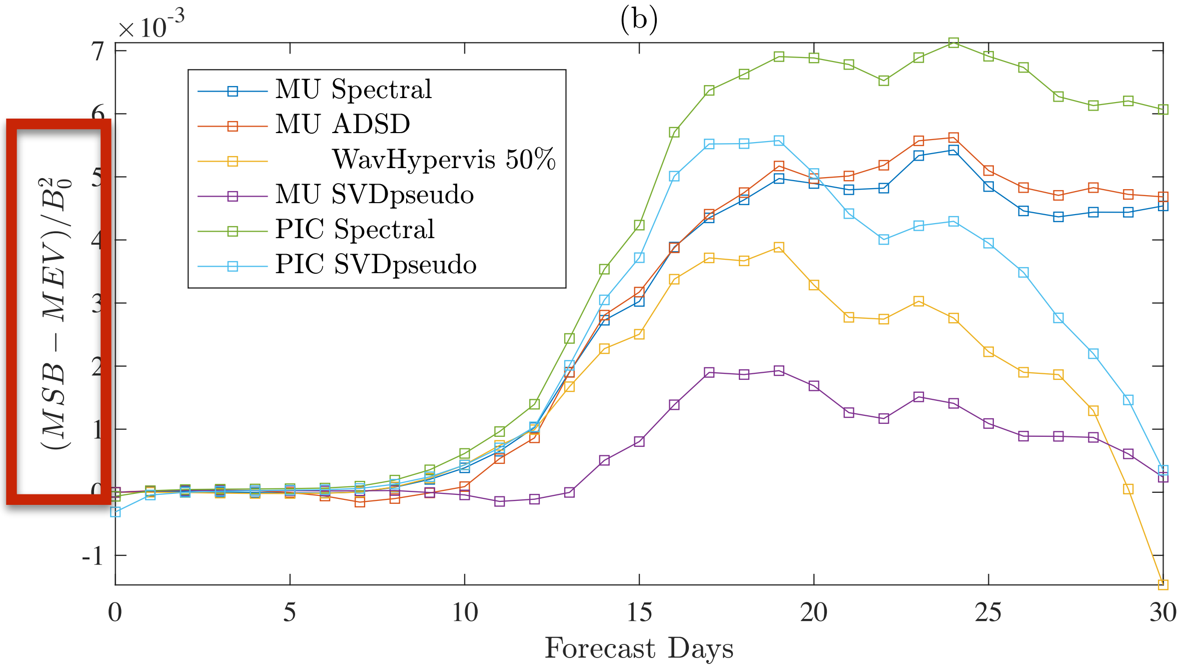
UQ metrics

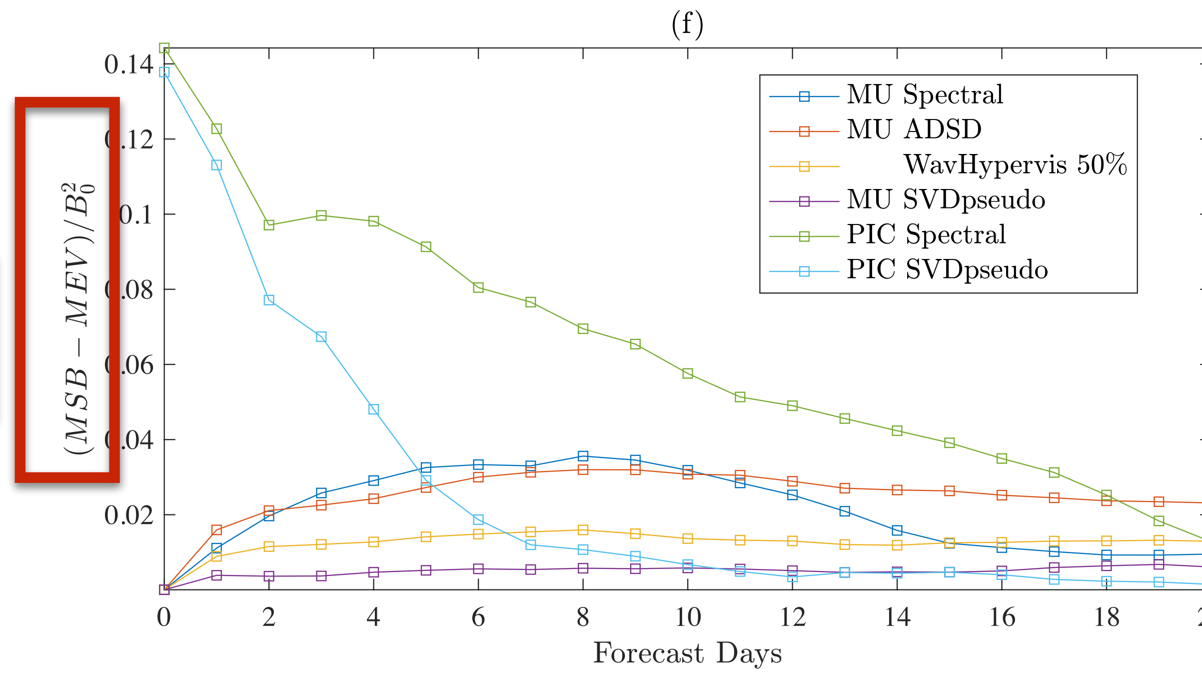
Metric	Meaning
RMSE	**Error** of ensemble members
Talagrand histogram (TH)	Capacity of the ensemble to **explore** all reference possible values
Bias^2-spread	Capacity of the ensemble to **explore** all reference possible values
CRPS	Point-wise distance between the ensemble CFD and the indicator function of the event
Energy Score (ES)	Generalized CRPS for multivariate ensemble



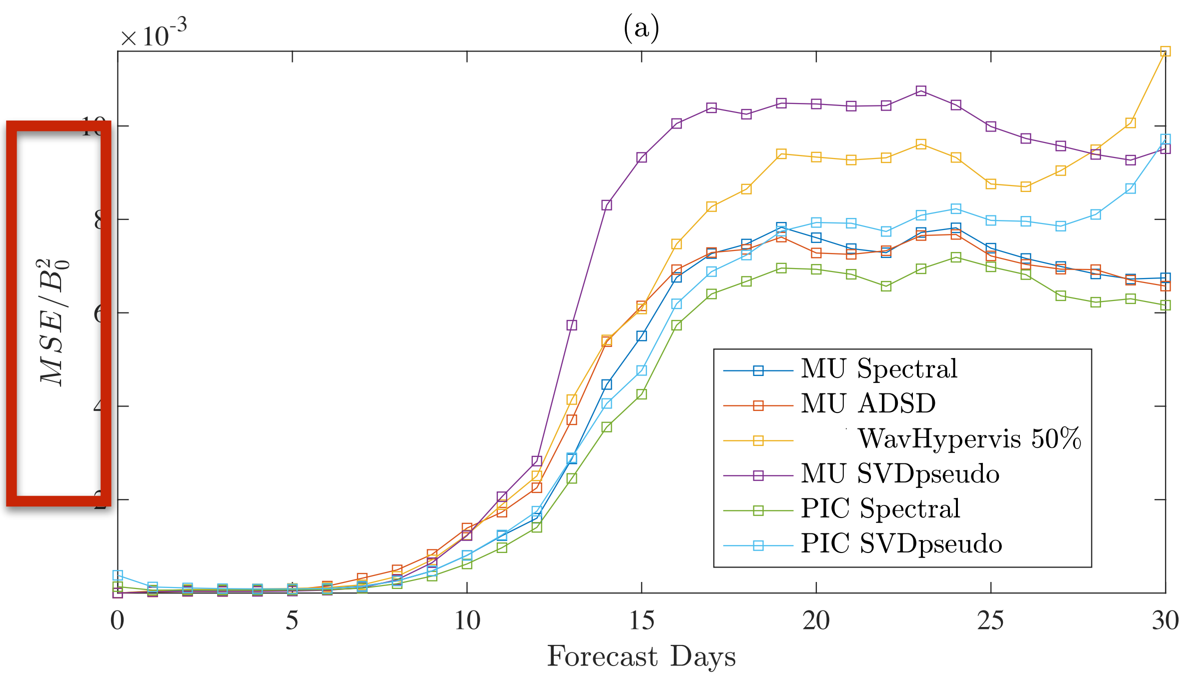
Spreading VS Errors



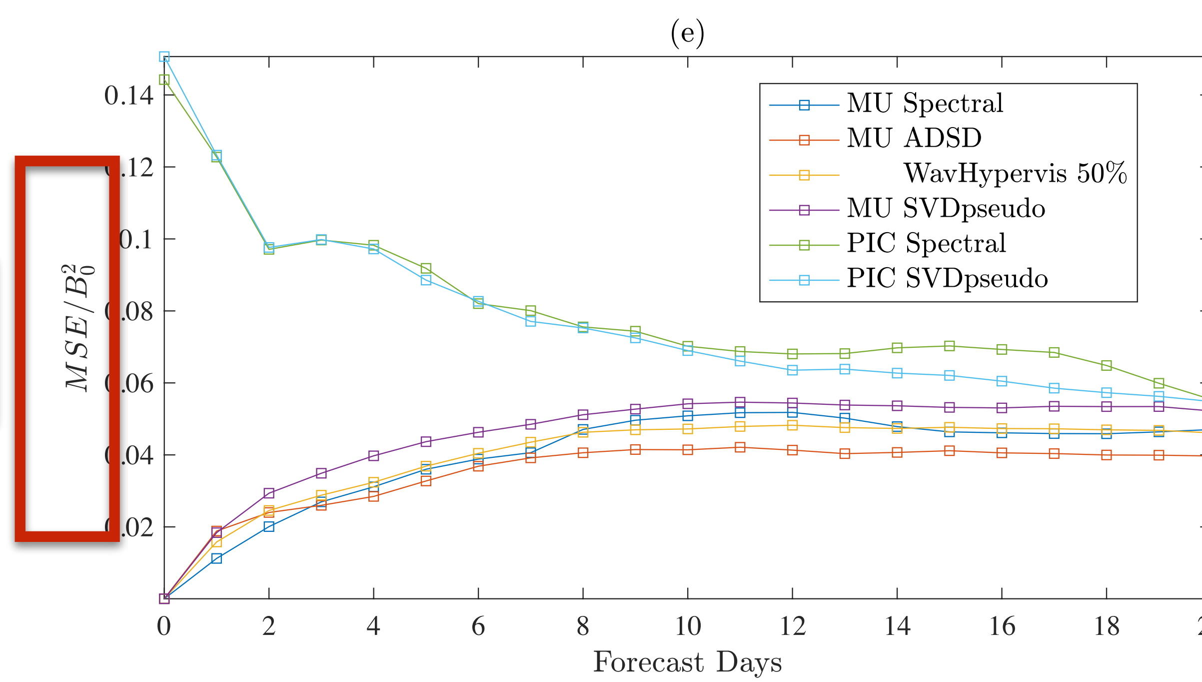


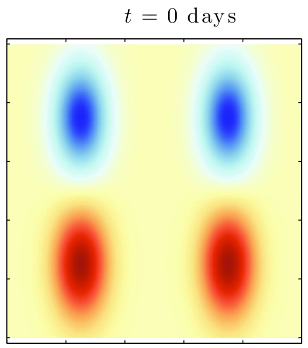


Spread



Error





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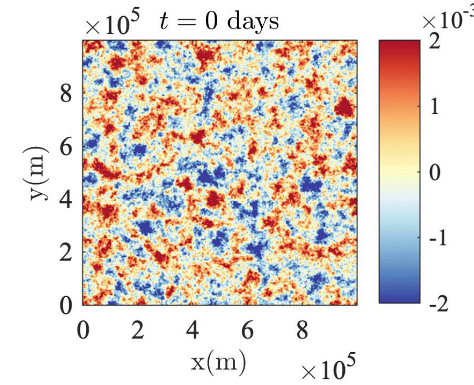
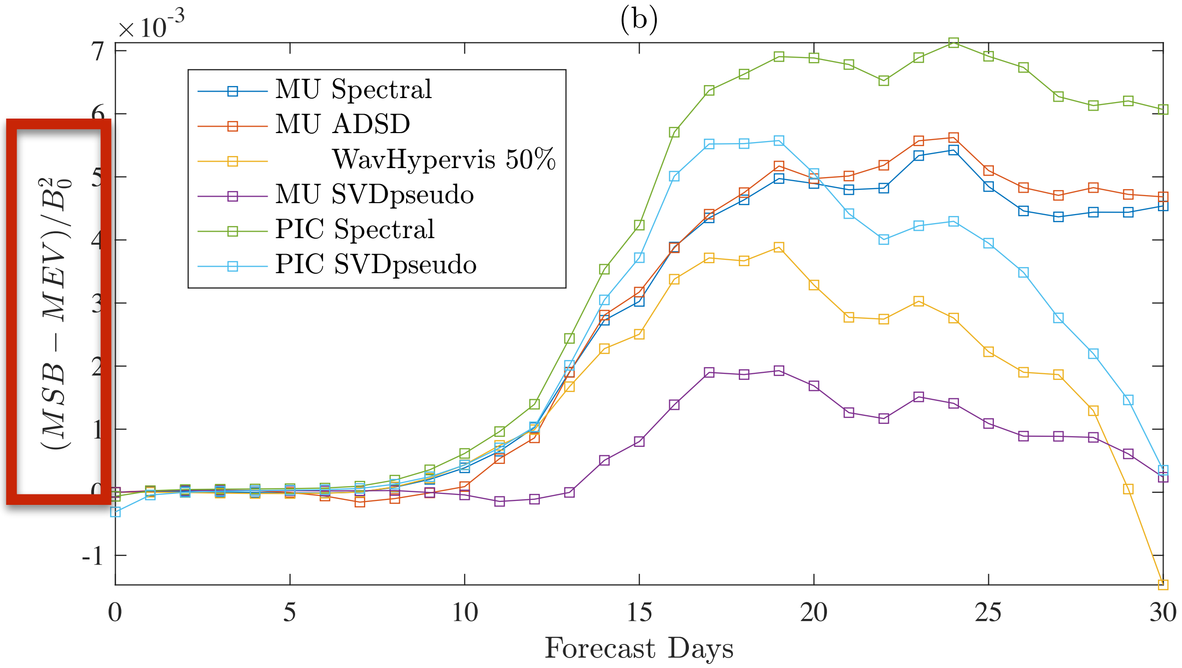
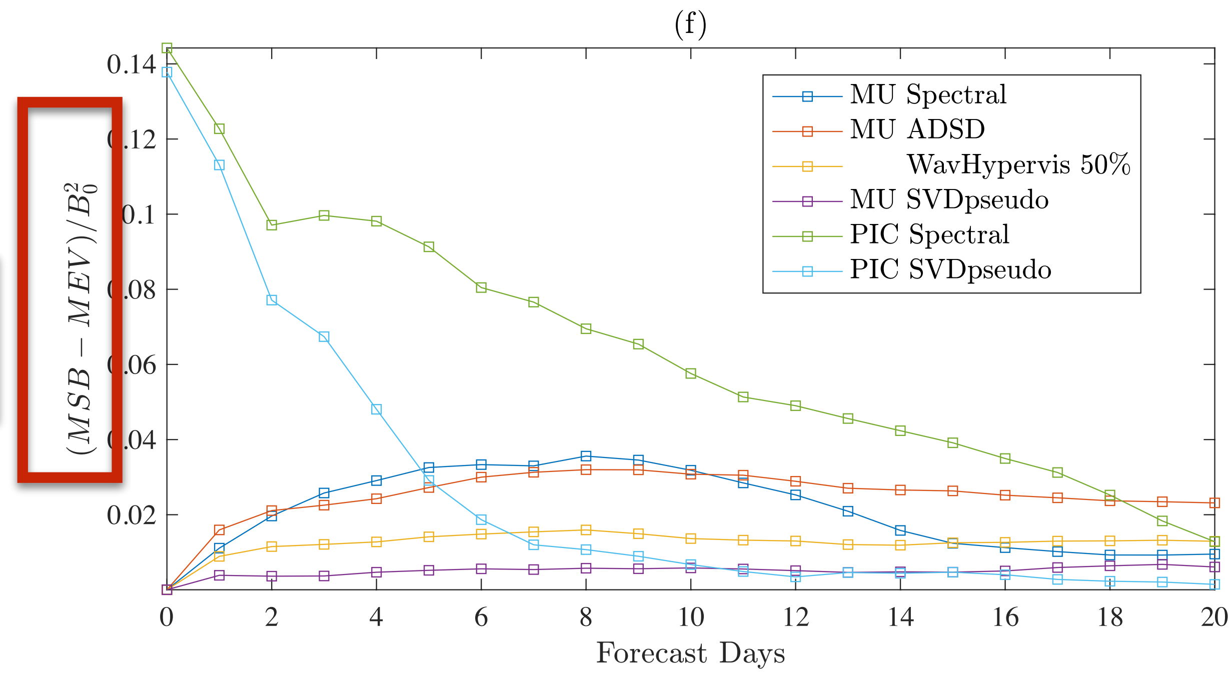
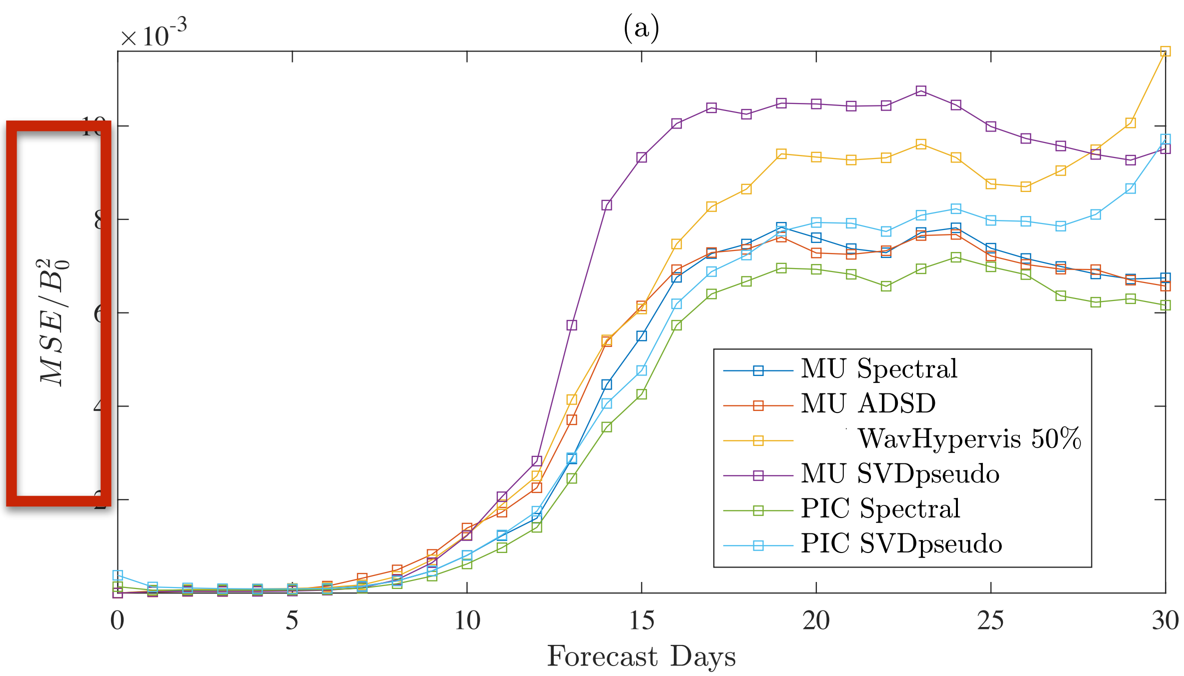


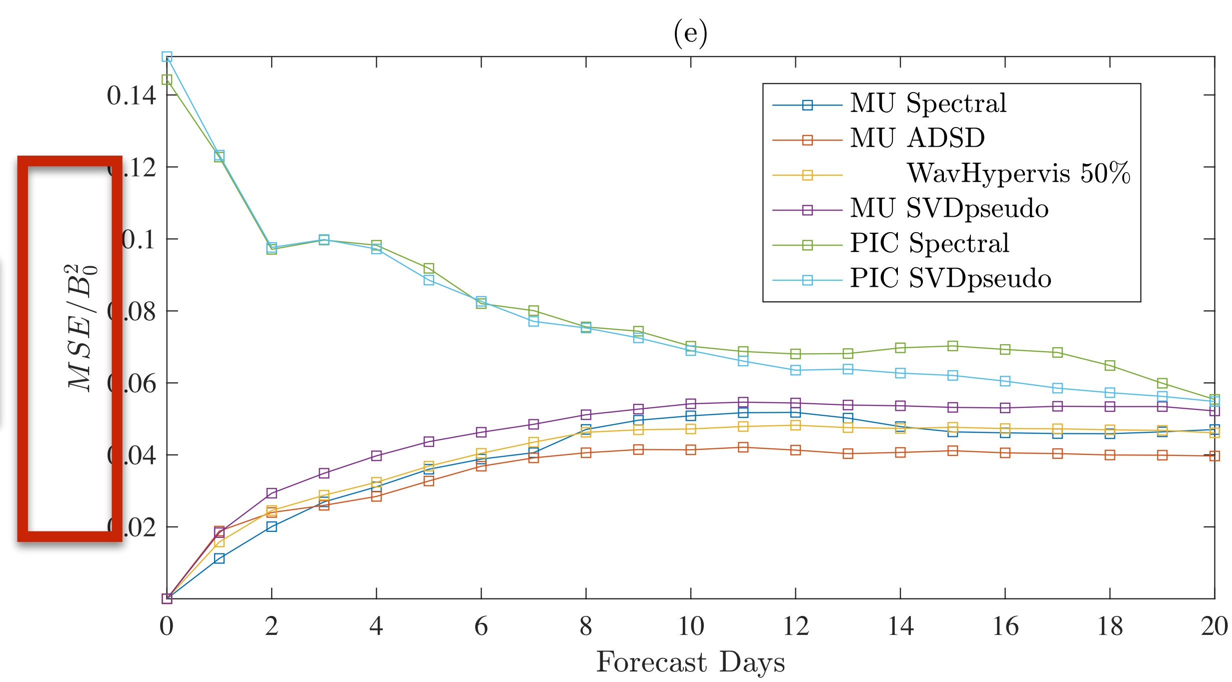


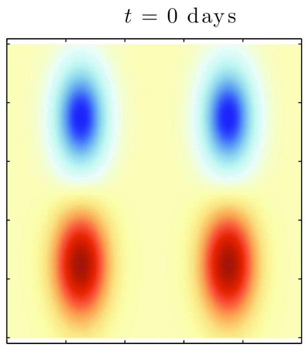


Spread

Error







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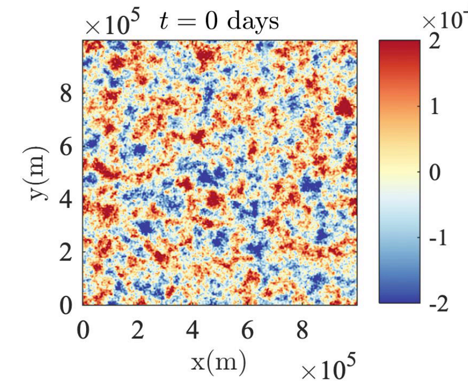
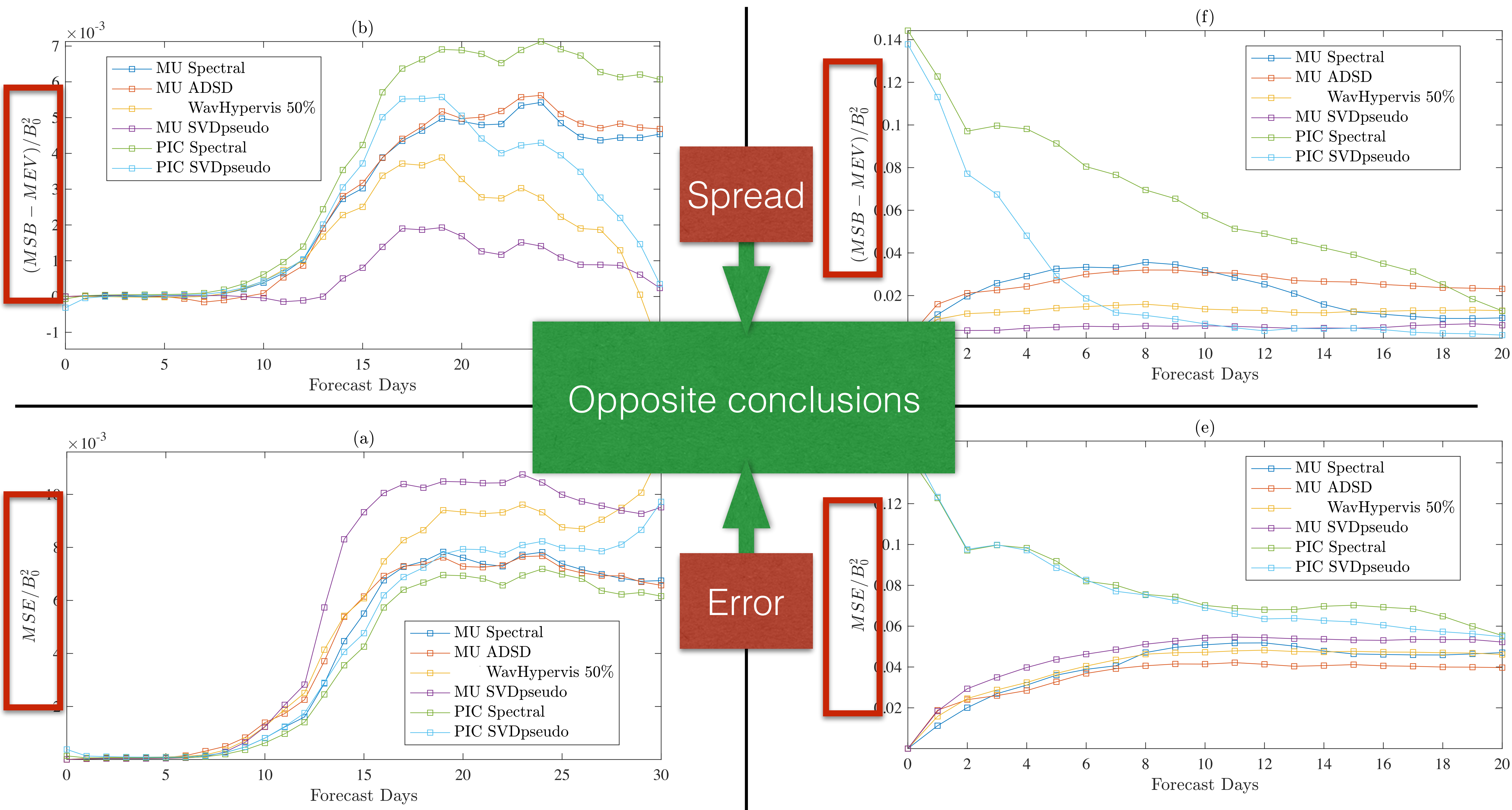
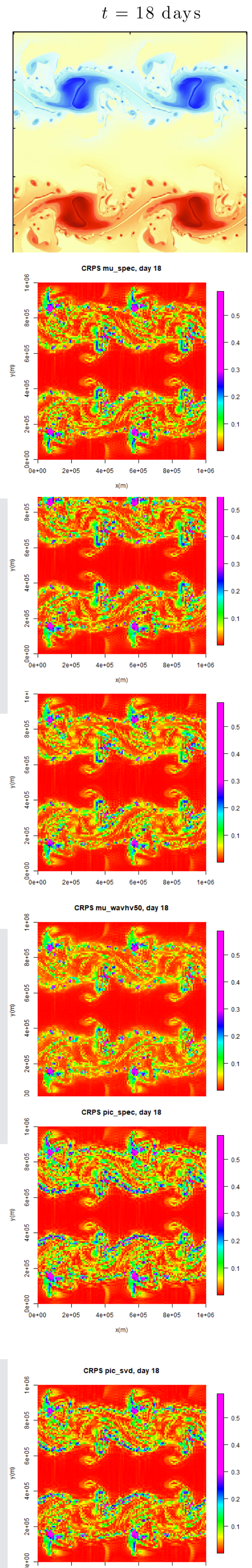
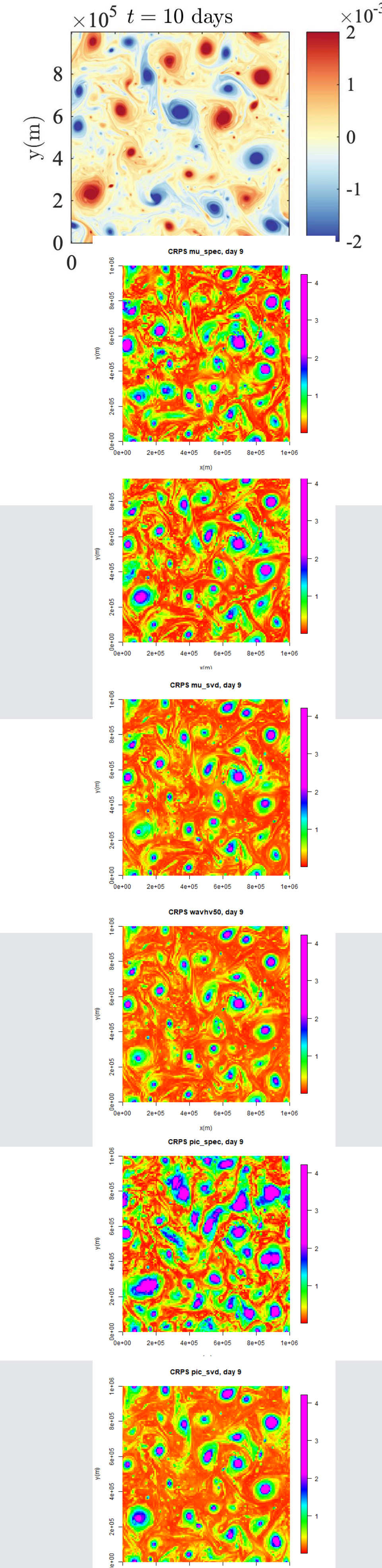






Ensemble point-wise skills : CRPS



MU Spec

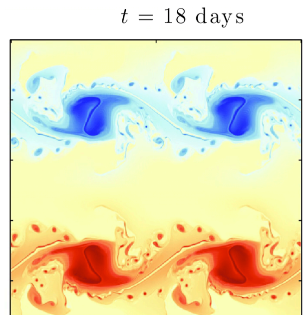
MU ADSD

MU SVD

WaveHyperv

PIC Spec

PIC SVD



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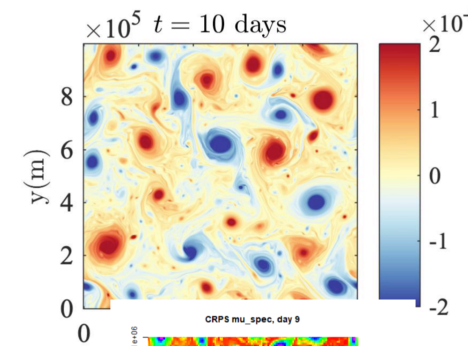

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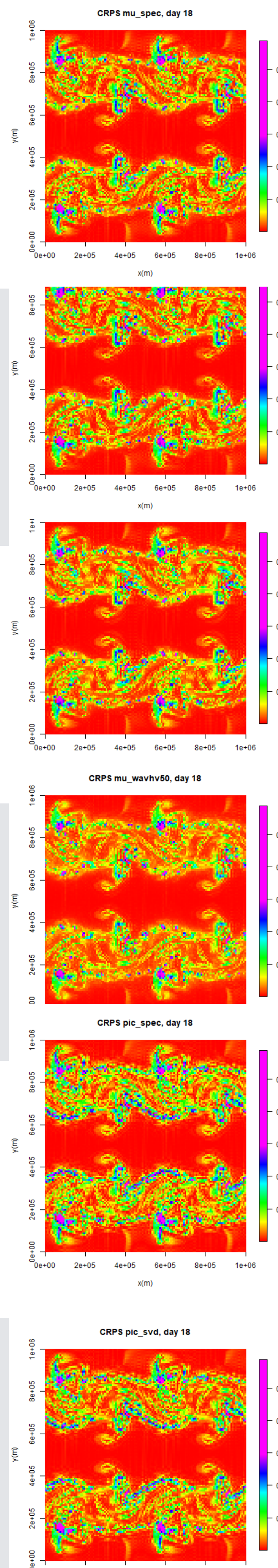

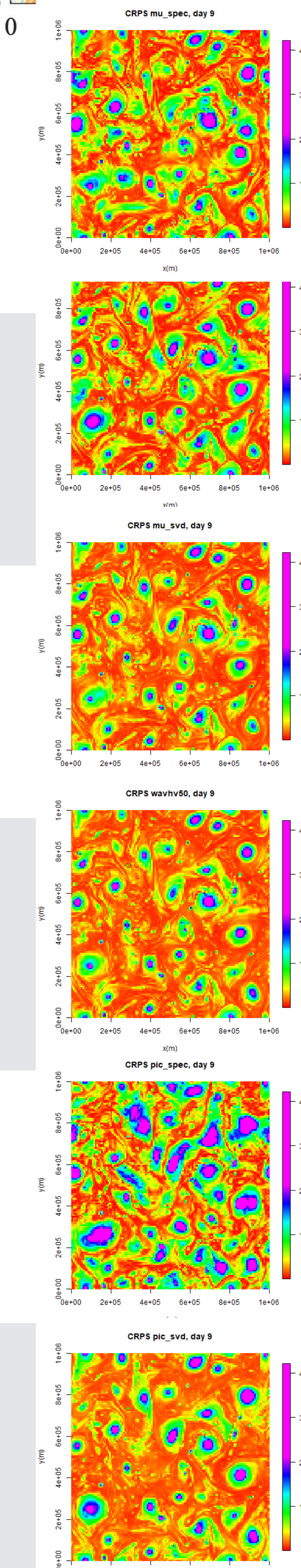

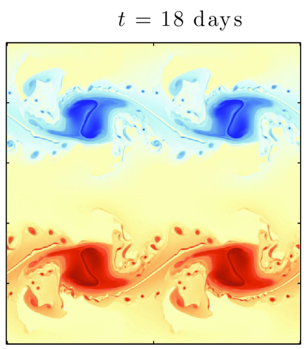

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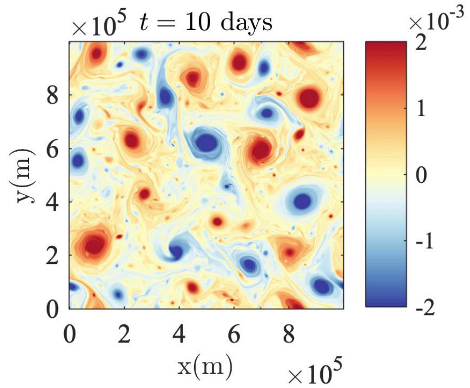

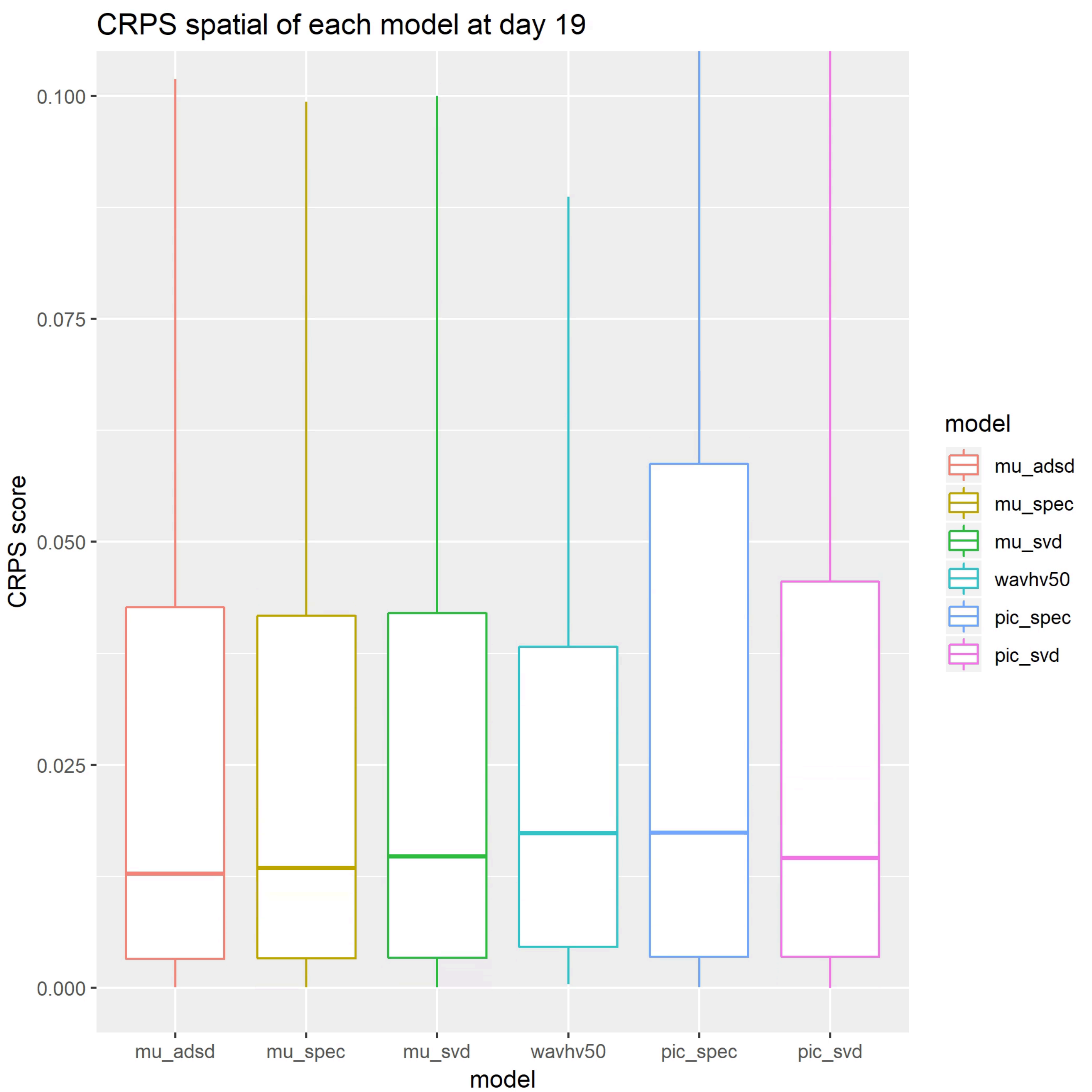

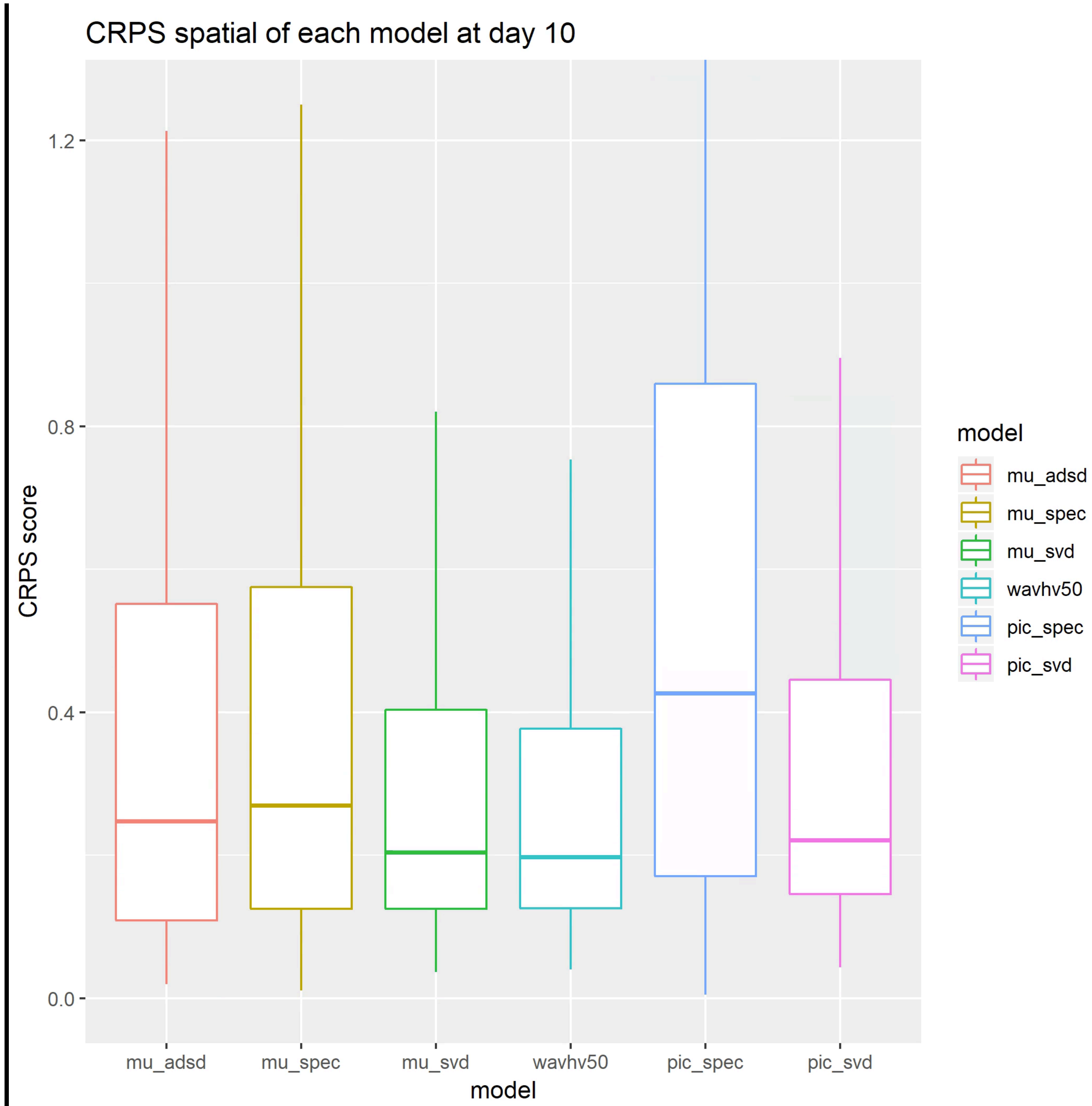

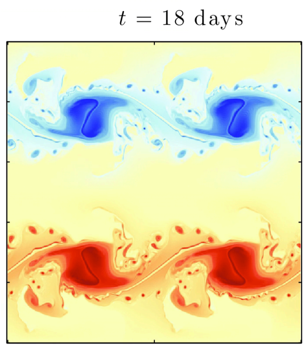

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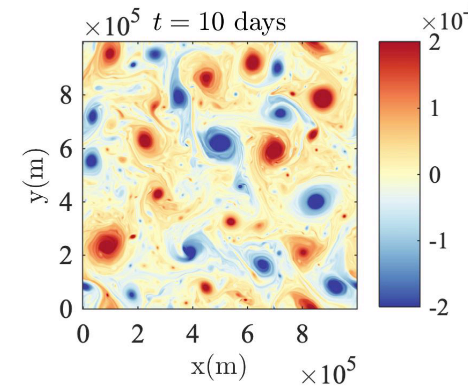

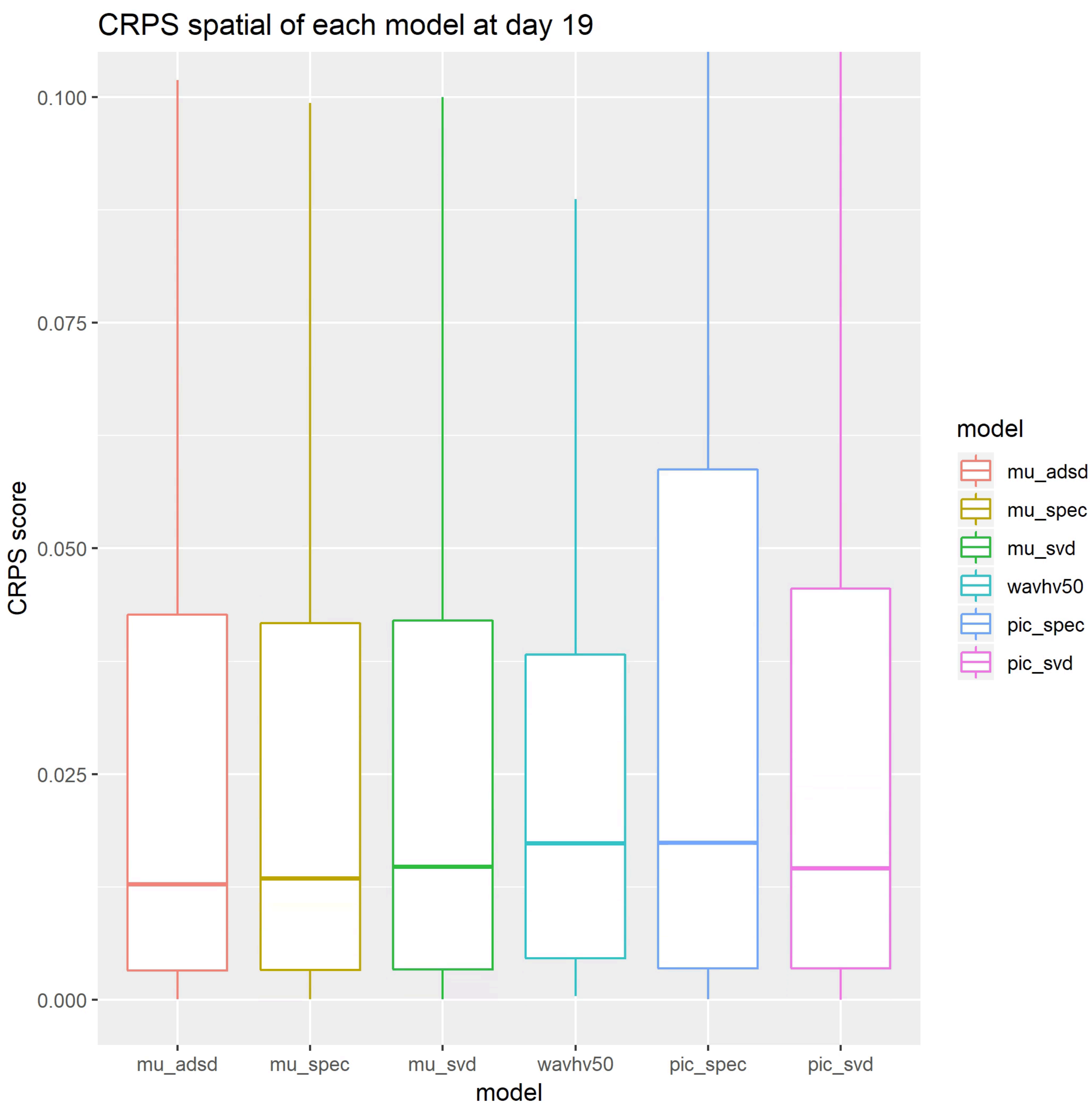

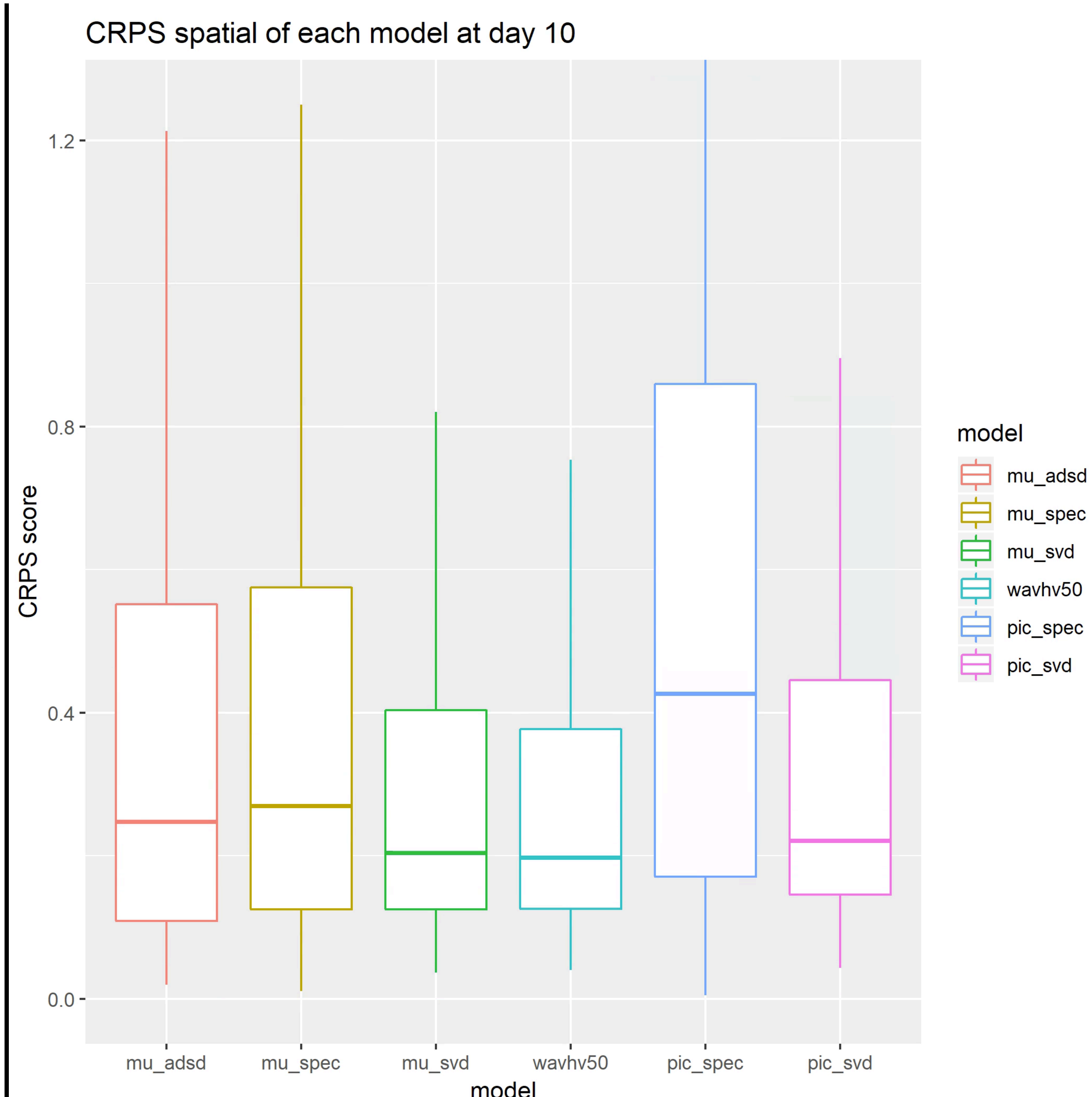

Conclusion

Conclusion

	RMSE (errors)	B^2-Var (spread)	CRPS (point-wise)	ES (global)
MU Spec	+	+	+	+
MU ADSD	++	+	+	+
MU SVD	-	++	+ +	++
WaveHyperv	-	+	++	+ +
PIC Spec	-	- -	- -	- -
PIC SVD	- -	-	+	+

Conclusion

	RMSE (errors)	B^2-Var (spread)	CRPS (point-wise)	ES (global)
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MU ADSD	++	+	+	+
MU SVD	-	++	+ +	++
WaveHyperv	-	+	++	+ +
~~PIC Spec~~	-	- -	- -	- -
~~PIC SVD~~	- -	-	+	+

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~~PIC SVD~~	- -	-	+	+

Conclusion

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MU SVD	-	++	+ +	++
WaveHyperv	-	+	++	+ +
~~PIC Spec~~	~~-~~	~~- -~~	~~- -~~	~~- -~~
~~PIC SVD~~	~~- -~~	~~-~~	~~+~~	~~+~~